Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Geometric Mean 3 −Equitable Labeling of Some Graphs

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ABSTRACT

In this paper we proved that $K_{mn}(m,n \geq 4)$ is not a geometric mean 3 −equitable graph, while caterpillar $S(x_1, x_2, \ldots, x_t)$, $C_n \odot tK_1 (t \geq 2)$ both are geometric mean 3 −equitable graphs. We also proved that $C_n \odot K_1$ is not geometric mean 3 −equitable graph, when $n \equiv 0 \pmod 3$, while it is geometric mean 3 −equitable graph, when $n \equiv 1, 2 \pmod 3$.

KEY WORDS : Caterpillar, corona graph, complete bipartite graph, cycle, star graph, geometric mean 3 −equitable graphs.

AMS subject classification number: 05C78.

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I. INTRODUCTION:

The concept of cordial and $3-$equitable labeling was introduced by Cahit[1, 2]. The labeled graphs have several application in the areas of radar, circuit design, cryptography etc. Mean cordial labeling was introduced by Ponraj, Sivakumar and Sundaram [6].

Geometric mean cordial labeling of graph was introduced by ChitraLakshmi and Nagarajan [3] and they have proved that P_n, $C_n\ (n \equiv 1,2\ (\text{mod } 3))$, $K_{1,n}$, $K_n(n \leq 2)$, $K_{2,n}(n \leq 2)$ are geometric mean cordial graphs and $K_n(n > 2)$, $K_{2,n}(n > 2)$, $K_{n,n}(n \geq 3)$, W_n are not geometric mean cordial graphs.

By survey of literature geometric mean cordial labeling defined by ChitraLakshmi and Nagarajan [3] its name should be geometric mean $3-$equitable labeling as they are using $e_f(0), e_f(1)$ and $e_f(2)$. For a (p,q)graph G, a function $f: V(G) \longrightarrow \{0,1,2\}$ with its induced edge labeling function $f^*: E(G) \longrightarrow \{0,1,2\}$ defined by $f^*(uv) = \left\lceil \sqrt{f(u)f(v)} \right\rceil$ is called geometric mean $3-$ equitable labeling if $|v_f(i) - v_f(j)|, |e_f(i) - e_f(j)| \in \{0,1\}$, where $v_f(x)$ and $e_f(x)$denotes the number of vertices and edges with x label respectively, where $x, i, j \in \{0,1,2\}$.

II. MAIN RESULTS :

***Theorem*$-$2.1:** Caterpillar $S(x_1, x_2, \dots, x_t)$ is a geometric mean $3-$equitable graph.

Proof :Let $V(S(x_1, x_2, \dots, x_t)) = \{v_i / 1 \leq i \leq t\} \cup \{v_{ij} / 1 \leq j \leq x_i, 1 \leq i \leq t\}$ and $E(S(x_1, x_2, \dots, x_t)) = \{v_i v_{i+1} / 1 \leq i \leq t-1\} \cup \{v_i v_{ij} / 1 \leq j \leq x_i, 1 \leq i \leq t\}$. It is obvious that $p = x_1 + x_2 + \cdots + x_t + t$ and $q = p - 1$ (as caterpillar is a tree). We redefine $V(S(x_1, x_2, \dots, x_t)) = \{u_k / 1 \leq k \leq p\}$ by taking $u_i = v_i (1 \leq i \leq t)$, $u_{t+j_1} = v_{1j_1} (1 \leq j_1 \leq x_1)$, $u_{t+x_{1}+j_2} = v_{2j_2} (1 \leq j_2 \leq x_2)$, ... , $u_{t+x_1+x_2+\cdots+x_{t-1}+j_t} = v_{tj_t} (1 \leq j_t \leq x_t)$.

Let $p_1 = \left\lceil \frac{p}{3} \right\rceil$, $p_2 = \left\lceil \frac{p-p_1}{2} \right\rceil$ and $p_3 = p - (p_1 + p_2)$.

Define $f: V(S(x_1, x_2, \dots, x_t)) \longrightarrow \{0,1,2\}$ as follows.

$$
\begin{aligned}
f(u_i) &= 1, && \text{when } 1 \leq i \leq p_1 \\
&= 2, && \text{when } p_1 + 1 \leq i \leq p_1 + p_2 \\
&= 0, && \text{when } p_1 + p_2 + 1 \leq i \leq p.
\end{aligned}
$$

Above defined labeling pattern give rise $v_f(0) = p_3$, $v_f(2) = p_2$, $v_f(1) = p_1$ and $e_f(1) = p_1 - 1$, $e_f(2) = p_2$, $e_f(0) = p_3$. In any case it is obvious that $|v_f(i) - v_f(j)|$, $|e_f(i) - e_f(j)| \in \{0,1\}, \forall i,j \in \{0,1,2\}$. Thus, $S(x_1, x_2, \dots, x_t)$ is a geometric mean $3-$equitable graph.

Theorem−2.2: $C_n \odot tK_1 (t \geq 2)$ is a geometric mean $3-$equitable graph.

Proof :Let $G = C_n \odot tK_1$, $V(G) = \{v_i/1 \leq i \leq n\} \cup \{v_{ij}/1 \leq j \leq t, 1 \leq i \leq n\}$ and $E(G) = \{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{v_1 v_n\} \cup \{v_i v_{ij}/1 \leq j \leq t, 1 \leq i \leq n\}$. Thus, $p = |V(G)| = (t+1)n = q$.

We redefine $V(G) = \{u_k/1 \leq k \leq p\}$ by taking $u_i = v_i (1 \leq i \leq n\}$, $u_{n+j_1} = v_{1j_1} (1 \leq j_1 \leq t)$, $u_{t+n_{+j_2}} = v_{2j_2} (1 \leq j_2 \leq t), \dots,$ $u_{n+(n-1)t+j_n} = v_{nj_n} (1 \leq j_n \leq t)$.

Define $f: V(G) \longrightarrow \{0,1,2\}$ as follows.

$$f(u_i) = 1, \quad \text{when} 1 \leq i \leq \left\lceil \frac{p}{3} \right\rceil$$

$$= 2, \quad \text{when } \left\lceil \frac{p}{3} \right\rceil + 1 \leq i \leq \left\lceil \frac{p-p_1}{2} \right\rceil + p_1$$

$$= 0, \quad \text{when } \left\lceil \frac{p-p_1}{2} \right\rceil + p_1 + 1 \leq i \leq p, \text{where } p_1 = \left\lceil \frac{p}{3} \right\rceil.$$

Above defined labeling pattern give rise $v_f(0) = \left\lfloor \frac{p}{3} \right\rfloor = \left\lfloor \frac{p-p_1}{2} \right\rfloor, v_f(2) = \left\lceil \frac{p-p_1}{2} \right\rceil, v_f(1) = p_1$ and $e_f(1) = p_1, e_f(2) = \left\lceil \frac{p-p_1}{2} \right\rceil, e_f(0) = \left\lfloor \frac{p}{3} \right\rfloor$. In any case it is observed that $|v_f(i) - v_f(j)|, |e_f(i) - e_f(j)| \in \{0,1\}, \forall i,j \in \{0,1,2\}$. Thus, G admits a geometric mean $3-$ equitable labeling and so, it is a geometric mean $3-$equitablegraph.

Theorem−2.3: $C_n \odot K_1$ is not geometric mean $3-$equitable graph, when $n \equiv 0 \pmod 3$ and it is geometric mean $3-$equitable graph, when $n \equiv 1, 2 \pmod 3$.

Proof :Let $H = C_n \odot K_1, V(H) = \{v_i, u_i/1 \leq i \leq n\}$ and $E(H) = \{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{v_1 v_n\} \cup \{u_i v_i/ 1 \leq i \leq n\}$. Thus $p = q = 2n$.

Case−I $\quad n \equiv 0 \pmod 3$. Take $n = 3t$.

If H admits any geometric mean $3-$equitable labeling f, then it is only possibly when $v_f(0) = v_f(1) = v_f(2) = 2t = \frac{p}{3} = \frac{2n}{3}$. Since, edge label 1 under f is only possible when its both the end vertices have label 1, under f, we must have $e_f(1) \le 2t-1$ and $\max\{e_f(0), e_f(2)\} \ge 2t+1$. Which gives either $|e_f(0) - e_f(1)| \ge 2$ or $|e_f(2) - e_f(1)| \ge 2$. This leads to a contradiction as f is a geometric mean $3-$equitable labeling for H. Thus, H can not admits any geometric mean $3-$equitable labeling. So, it is not a geometric mean $3-$equitable graph, when $n \equiv 0 \pmod 3$.

Case−II $\quad n \equiv 1, 2 \pmod 3$.

Let $p_1 = \left\lceil \frac{p}{3} \right\rceil, p_2 = \left\lceil \frac{p-p_1}{2} \right\rceil$ and $p_3 = p - (p_1 + p_2)$.

Define $f: V(H) \longrightarrow \{0,1,2\}$ as follows.

$$f(v_i) = 1, \quad \forall\, 1 \le i \le p_1$$
$$= 2, \quad \forall p_1 + 1 \le i \le n;$$
$$f(u_i) = 0, \quad \forall\, 1 \le i \le p_2$$
$$= 2, \quad \forall p_2 + 1 \le i \le n.$$

Above defined labeling pattern give rise $v_f(1) = p_1, v_f(0) = p_2,\ v_f(2) = p_3$ and $e_f(1) = p_1 - 1, e_f(0) = p_2, e_f(2) = p_3 + 1$. Since $p_3 = p_1 - 1$, $|p_i - p_j| \in \{0,1\}, \forall i \in \{0,1,2\}$, we must get $|v_f(i) - v_f(j)|, |e_f(i) - e_f(j)| \in \{0,1\}, \forall i, j \in \{0,1,2\}$. Thus, H admits geometric mean $3-$equitable labeling f and so, it is a geometric mean $3-$equitable graph, when $n \equiv 1, 2 \pmod 3$.

Theorem−2.4: $K_{m,n}$ is not a geometric mean $3-$equitable graph, when $m, n \ge 4$.

Proof : Let $V(K_{m,n}) = M \cup N$. We take $M = \{u_1, u_2, \dots, u_m\}$ and $N = \{v_1, v_2, \dots, v_n\}$ both are two partite sets of $K_{m,n}$. Let $E(K_{m,n}) = \{u_i v_j / 1 \le i \le m,\ 1 \le j \le n\}$. It is obvious that $p = m + n$ and $q = mn$.

Let $f: V(K_{m,n}) \longrightarrow \{0,1,2\}$ be any vertex labeling. To make geometric mean $3-$equitable labeling f for $K_{m,n}$, we have to choose $\max\{v_f(0), v_f(1), v_f(2)\} - min\{v_f(0), v_f(1), v_f(2)\} \in \{0,1\}$. Let $max\{v_f(0), v_f(1), v_f(2)\} = t$.i.e. $t = \left\lceil \frac{p}{3} \right\rceil$. Since, edge label 1 under f is only possible when its both the end vertices have label 1 under f. We shall take the following three cases to compute $e_f(1)$ for $K_{m,n}$.

Case−I $\frac{t}{2} \geq \min\{m,n\} = n$ (say).

It is observe that $e_f(1) \leq (t-n) \cdot n = tn - n^2$.

First we shall prove here $tn - n^2 < \left\lfloor \frac{mn}{3} \right\rfloor$. Suppose not if possible.

i.e. $tn - n^2 \geq \left\lfloor \frac{mn}{3} \right\rfloor$

$\Rightarrow 3tn - 3n^2 \geq mn$

$\Rightarrow (3(t-n) - m)n \geq 0$

$\Rightarrow (3(t-n) - m) \geq 0$

$\Rightarrow 3t - 3n \geq m$

$\Rightarrow 3t - 2n \geq m + n = p$

$\Rightarrow p + 2 - 2n \geq p$

$\Rightarrow 2 - 2n \geq 0$,

Which is impossible as $n \geq 4$ and so, $tn - n^2 \geq \left\lfloor \frac{mn}{3} \right\rfloor$ can not holds. Thus, $e_f(1) \leq tn - n^2 < \left\lfloor \frac{mn}{3} \right\rfloor$.

Case−II $\frac{t}{2} < \min\{m,n\} = n$ (say) and $\frac{t}{2} < \frac{m}{3}$.

$\Rightarrow \left(\frac{t}{2}\right)^2 < \frac{mn}{3}$

$\Rightarrow e_f(1) < \left(\frac{t}{2}\right)^2 \leq \left\lfloor \frac{mn}{3} \right\rfloor$

$i.e. e_f(1) < \left\lfloor \frac{mn}{3} \right\rfloor$.

Case−III $\frac{t}{2} < \min\{m,n\} = n$ (say) and $\frac{t}{2} \geq \frac{m}{3}$.

$\Rightarrow 3t \geq 2m$

$\Rightarrow m + n + 2 \geq 2m$

$\Rightarrow m \leq n + 2.$

Now we see that $t \leq \frac{m+n+2}{3} \leq \frac{2n+4}{3} \leq n$, as $n \geq 4$

$\Rightarrow t \leq m, n$

$\Rightarrow t^2 \leq mn$

$\Rightarrow \frac{t^2}{4} \leq \frac{mn}{4} < \frac{mn}{3}$

$\Rightarrow e_f(1) \leq \frac{t^2}{4} < \left\lfloor \frac{mn}{3} \right\rfloor.$

Thus, in any case $e_f(1) < \left\lfloor \frac{mn}{3} \right\rfloor$, which gives $\max\{e_f(0), e_f(2)\} \geq \left\lfloor \frac{mn}{3} \right\rfloor + 2$.

$\therefore \max\{e_f(0), e_f(2)\} - e_f(1) \geq 2$

$\therefore f$ can not be a geometric mean $3-$equitable labeling for $K_{m,n}$. Hence, $K_{m,n}$ is not a geometric mean $3-$equitable graph, when $m, n \geq 4$.

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