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Soret and Dufour Effects on Flow, Heat and Mass Transfer due to Indirect Natural Convection

Konwar Hemanta*

Department of Mathematics, Kohima Science College, Jotsoma, India.
Email Id: hemanta.konwar@gmail.com

ABSTRACT

In this paper, an investigation has been made to study the effects of Soret and Dufour on steady, laminar boundary layer flow over a horizontal uniformly heated semi-infinite plate due to indirect natural convection. The governing nonlinear partial differential equations are transformed into a system of coupled ordinary differential equations by using suitable substitutions and are solved numerically by using MATLAB's built-in-solver bvp4c. The results of the velocity, temperature and concentration fields are presented graphically and discussed for Soret and Dufour number. Also, local skin friction, local heat and mass transfer coefficients at the surface are obtained and tabulated.

KEYWORDS: horizontal plate, indirect natural convection, Soret, Dufour, bvp4c.

***Corresponding Author:**

Konwar Hemanta*

Department of Mathematics,

Kohima Science College, Jotsoma, India.

Email Id: hemanta.konwar@gmail.com

INTRODUCTION:

The flow over a horizontal plate is a classical problem in fluid mechanics. Fluid motion driven by an indirectly induced pressure gradient normal to the direction of density potential is called indirect natural convection. The indirect natural convection from a heated horizontal plate in a fluid has been investigated experimentally in recent years due to its wide range of applications in applied sciences and engineering such as, cooling of electronic equipments, cooling of nuclear reactors, extraction geothermal energy, ground water flow, etc.

When a low-viscous fluid past a hot vertical plate then convection takes place in the boundary layer originating at the lower edge of the plate and heat transferred from the plate to the fluid leads to decrease in density of the fluid near the surface of the plate due to rise in temperature and as a result gaining buoyancy force moves the fluid upward along the plate.

But in case of heated semi-infinite horizontal plate facing upward the buoyancy force has no component along its length. In this case, heat is absorbed by the surrounding fluid progressively, thus inducing a horizontal temperature gradient within the fluid, which in turn give rise to a favourable pressure gradient that leads to the formation of boundary layer flow at the surface of the plate due to indirect natural convection provided that a suitably defined Grashof number is large. Mathematically, above the surface of the horizontal plate, the temperature is everywhere T_∞ so that, as in the static field, there exist a pressure distribution having pressure gradient $|\partial p/\partial y| = \rho_\infty g$ where origin is taken at one of the leading edge. In the boundary region adjacent to the plate dimensional temperature T_w is larger than T_∞ and so the density ρ of the fluid is lower than ρ_∞. Decreased in pressure gradient $|\partial p/\partial y| = \rho g < \rho_\infty g$ in the boundary layer region leads to a pressure drop in the x-direction. This reduced pressure gradient in x-direction is the origin of the indirect natural convection flow parallel to the plate and takes place at large Grashof number.

It was first shown by Stewartson[1] that such an indirect natural convection flow exists on the upper side of a horizontal plate when temperature of the plate is more than the surroundings. Rotem and Claassen[2] studied numerically natural convection above unconfined horizontal surfaces. Goldstein et al.[3] investigated experimentally heat and mass transfer adjacent to horizontal plate. Al-arabi and El-Riedy[4] have discussed natural convection heat transform an isothermal horizontal plates of different shapes. Sparrow and Carlson[5] discussed local and average natural convection Nusselt number for a uniformly heated horizontal plate. Noshadi and Schneider[6] investigated natural convection flow far from a horizontal plate. Schlichting and Gersten[7] have presented a similarity solution for horizontal semi-infinite plates for constant wall temperature.

The study of heat and mass transfer in boundary layer flow has great importance in fluid dynamics. It has been experienced that when heat and mass transfer occurs simultaneously the driving potential is of more complicated in nature, as energy flux can be generated not only by temperature gradient but also by concentration gradient as well.

Generally, it is known that heat and mass fluxes are generated from temperature and concentration gradient, respectively. Mass flux due to the temperature gradient is known as Soret effect or thermo-diffusion and heat flux that occurs in a chemically reactive system due to the concentration gradient is called Dufour effect or diffusion-thermo. Usually these effects are important under a large temperature and concentration gradient where more than one chemical species are present in fluid and each species has its own diffusion velocity. Soret and Dufour effects are important due to a wide range of applications such as the solidification of binary alloys, separation of mixtures of gases with light molecular elements, isotope separation, pollution control, etc. Eckert and Drake[8] first showed that though Soret and Dufour effects are smaller order of magnitude, but cannot be neglected. Dursunkaya and Worek[9] studied boundary layer flow considering Soret and Dufour effects from vertical surface. Kafoussias and Williams[10] investigated the effects of Soret and Dufour on mixed convection and mass transfer laminar boundary layer flow over a vertical flat plate. Joly et al.[11] analyzed the thermal and solutal effects on natural convection in a vertical enclosure. Postelnicu[12] studied heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects.

Abreu et al.[13] studied Soret and Dufour effects in boundary layer flows.

In this paper an investigation is made by considering Soret and Dufour effects due to indirect natural convection flow as their effects are important in transport phenomena.

MATHEMATICAL FORMULATION:

Consider a steady, laminar, boundary layer flow of a thermally and electrically conducting, chemically reacting, incompressible viscous Newtonian fluid over a uniformly heated semi-infinite horizontal flat plate facing upward with a single leading edge. The plate is maintained at uniform surface temperature T_w while the quiescent fluid is maintained at a lower temperature $T_\infty (<T_w)$. The physical model and the coordinate system are shown in the Fig.1. We take origin at one end of the plate, x-axis is along the surface of the plate and y-axis is normal to it.

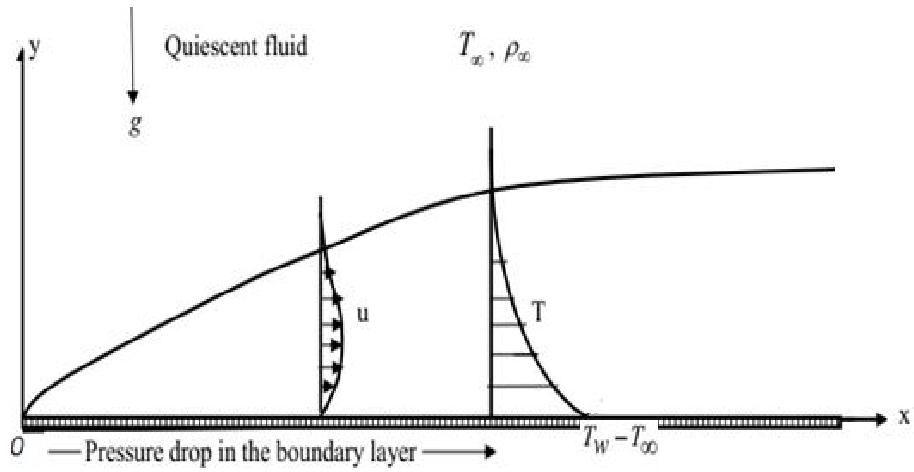

Fig.1: Formation of a pressure gradient $\partial p / \partial x$ in the boundary layer due to indirect natural convection.

Using Boussinesq approximation the continuity, momentum equations in *x* and *y*-directions, energy equation and concentration distribution equation in the boundary layer region in dimensional form are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$0=-\frac{\partial p}{\partial y}+\rho g\beta_T\left(T-T_\infty\right)+\rho g\beta_C\left(C-C_\infty\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Dk_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \tag{4}$$

and

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+\frac{Dk_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

with the boundary conditions

$$u=0, v=0, T=T_w, C=C_w \quad \text{at} \quad y=0 \tag{6}$$

and

$$u\rightarrow 0, T\rightarrow T_\infty, C\rightarrow C_\infty, p\rightarrow p_\infty \quad \text{at} \quad y\rightarrow\infty \tag{7}$$

Introducing the following non-dimensional quantities,

$$\bar{x}=\frac{x}{l},\ \bar{y}=\frac{y}{l}(Gr_L)^{1/5},\ \bar{u}=\frac{lu}{\upsilon}(Gr_L)^{-2/5},\ \bar{p}=\frac{(p-p_\infty)l^2}{\rho\upsilon^2}(Gr_L)^{-4/5},\ \bar{v}=\frac{lv}{\upsilon}(Gr_L)^{-1/5},\ V_{IN}=\left(gl^{1/2}\upsilon^{1/2}\beta_T\Delta T\right)^{2/5},$$

$$T^*=\frac{T-T_\infty}{\Delta T},\ C^*=\frac{C-C_\infty}{\Delta C},\ Pr=\frac{\upsilon}{\alpha},\ Sc=\frac{\upsilon}{D},\ Gr_L=\frac{g\beta_T l^3\Delta T}{\upsilon^2},\ D_f=\frac{Dk_T\Delta C}{c_s c_p\upsilon\Delta T},\ S_T=\frac{Dk_T\Delta T}{T_m\upsilon\Delta C},\ N=\frac{\beta_C\Delta C}{\beta_T\Delta T},$$

$$\Delta T=T_w-T_\infty,\ \Delta C=C_w-C_\infty \qquad (8)$$

into dimensional boundary layer Eqs.(1)-(7) we get non-dimensional boundary layer equations as follows:

$$\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}=0, \qquad (9)$$

$$\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+v\frac{\partial\bar{u}}{\partial\bar{y}}=-\frac{\partial\bar{p}}{\partial\bar{x}}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2} \qquad (10)$$

$$0=-\frac{\partial\bar{p}}{\partial\bar{y}}+T^*+NC^* \qquad (11)$$

$$\bar{u}\frac{\partial T^*}{\partial\bar{x}}+\bar{v}\frac{\partial T^*}{\partial\bar{y}}=\frac{1}{Pr}\frac{\partial^2T^*}{\partial\bar{y}^2}+D_f\frac{\partial^2C^*}{\partial\bar{y}^2} \qquad (12)$$

and

$$\bar{u}\frac{\partial C^*}{\partial\bar{x}}+v\frac{\partial C^*}{\partial\bar{y}}=\frac{1}{Sc}\frac{\partial^2C^*}{\partial\bar{y}^2}+S_T\frac{\partial^2T^*}{\partial\bar{y}^2} \qquad (13)$$

along with the non-dimensional boundary conditions

$$\bar{u}=0,\bar{v}=0,T^*=1,C^*=1\ \text{at}\ \bar{y}=0 \qquad (14)$$

and

$$\bar{u}\to 0,T^*\to 0,C^*\to 0,\bar{p}\to 0\ \text{at}\ \bar{y}\to\infty \qquad (15)$$

Note that the powers in Eq. (8) are so chosen that the continuity equation, a viscous term in the *x*-momentum equation as well as the pressure and buoyancy terms in the *y*-momentum equation remain the same after the transformation in the limit $Gr_L\to\infty$.

METHOD OF SOLUTION:

We now introduce non-dimensional stream function $\bar{\psi}\left(\bar{x},\bar{y}\right)$ such that

$$\bar{u}=\partial\bar{\psi}/\partial\bar{y}\ \text{and}\ \bar{v}=-\partial\bar{\psi}/\partial\bar{x} \qquad (16)$$

where functional forms for $\bar{\psi},\ \bar{p},\ T^*$ and C^* in terms of similarity variable η are defined by

$$\bar{\psi}=\bar{x}^{3/5}f(\eta),\ \eta=\bar{y}\,\bar{x}^{-2/5},\ \bar{p}=\bar{x}^{2/5}h(\eta),\ T^*=\theta(\eta)\ \text{and}\ C^*=\varphi(\eta). \qquad (17)$$

The non-dimensional equation of continuity Eq.(9) is identically satisfied when $\bar{u}$ and $\bar{v}$ are expressed in terms of $\bar{\psi}(\bar{x},\bar{y})$ as defined by the equation Eq.(16).

Substituting similarity transformations given by Eq.(17) into the boundary layer Eqs.(10)-(15) we get the following ordinary differential equations:

$$f''' + \frac{3}{5}ff'' - \frac{1}{5}(f')^2 = \frac{2}{5}(h - \eta h') \tag{18}$$

$$h' = \theta + N\varphi \tag{19}$$

$$\theta'' + \frac{3}{5}Prf\theta' + D_f Pr\varphi'' = 0 \tag{20}$$

and

$$\varphi'' + \frac{3}{5}Scf\varphi' + S_T Sc\theta'' = 0 \tag{21}$$

along with the new boundary conditions

$$f = 0,\ f' = 0,\ \theta = 1,\ \varphi = 1 \quad \text{when } \eta = 0 \tag{22}$$

and

$$f' \to 0,\ \theta \to 0,\ \varphi \to 0,\ h \to 0 \ \text{ when } \eta \to \infty \tag{23}$$

where prime denotes the differentiation with respect to η. The nonlinear coupled ordinary differential equations Eqs.(18)-(21) are solved numerically by using MATLAB's built in solver bvp4c by taking into consideration of boundary conditions Eqs.(22)-(23).

RESULTS AND DISCUSSIONS:

In order to get a physical insight into the problem, a representative set of numerical results are shown graphically for Soret and Dufour numbers.

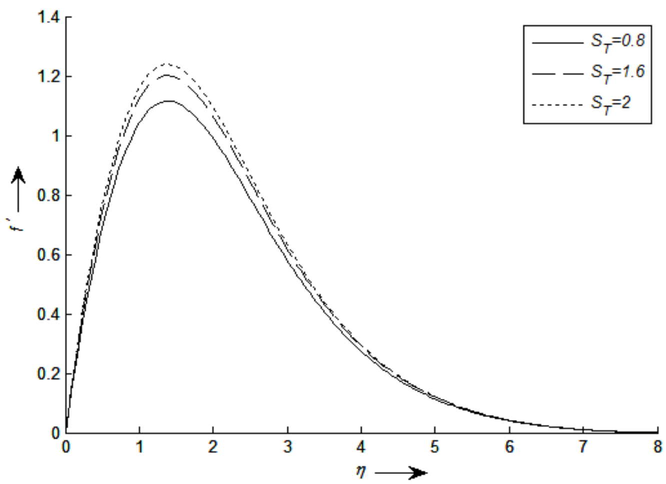

Fig.2: Horizontal velocity profile for different values of S_T

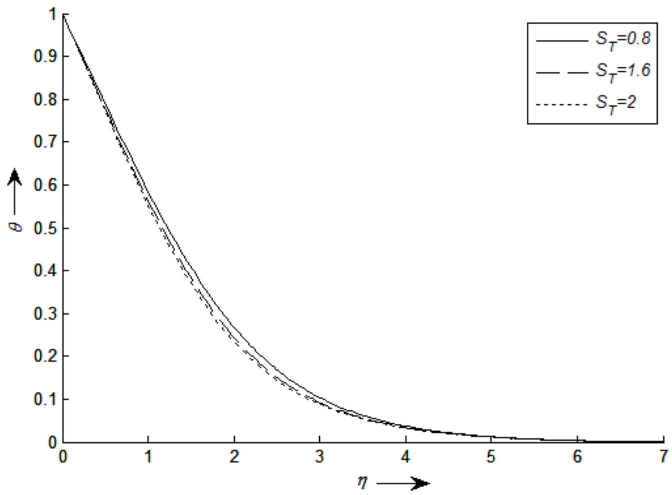

Fig.3: Temperature profile for different values of S_T

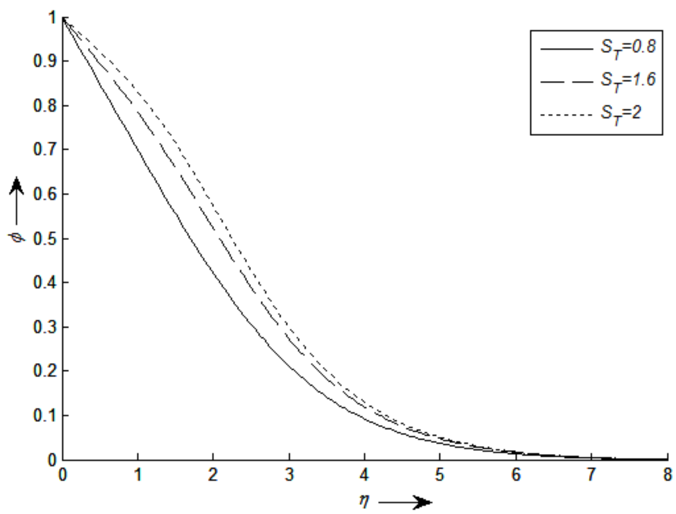

Fig.4: Concentration profile for different values of S_T

Figs.2, 3 and 4 depict the Soret effect on dimensionless horizontal velocity, temperature and concentration of the fluid respectively for values of $N=1, Pr=0.72, Sc=0.57, D_f=0.2$.

It is clear from Fig.2 that magnitude of horizontal velocity component increases asymptotically from minimum to maximum value within a thin boundary layer near the plate and then decreases exponentially towards the upper edge of the boundary layer as η increases. Moreover, increasing values of S_T increases horizontal velocity component of the fluid. Fig.3 depicts that increase in the values of S_T decreases the temperature of the fluid. It is because of the fact that larger the Soret number lower the thermal boundary layer thickness. It has been observed from the Fig.4 that concentration of species of the fluid is high at the surface of the plate and low towards the upper edge of the boundary layer as η increases. Moreover, concentration of species of the fluid increases at any point in the boundary layer as the value of S_T increases. It is true because increase in Soret number raises temperature gradient which causes high mass flux and as a result concentration of species of the fluid increases in the boundary layer.

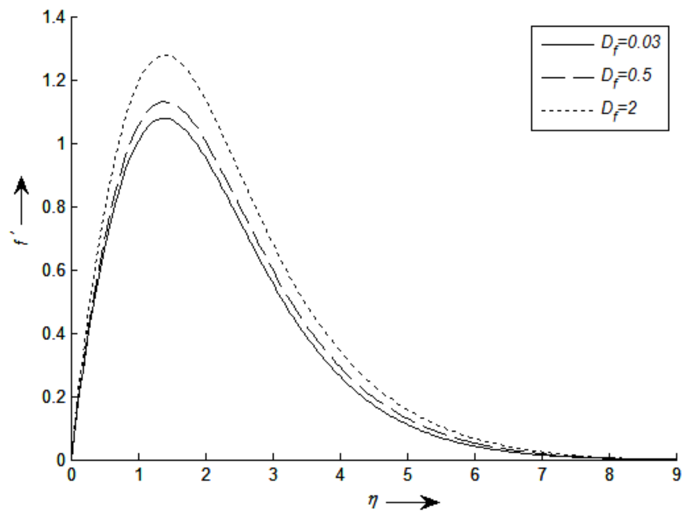

Fig.5: Horizontal velocity profile for different values of D_f

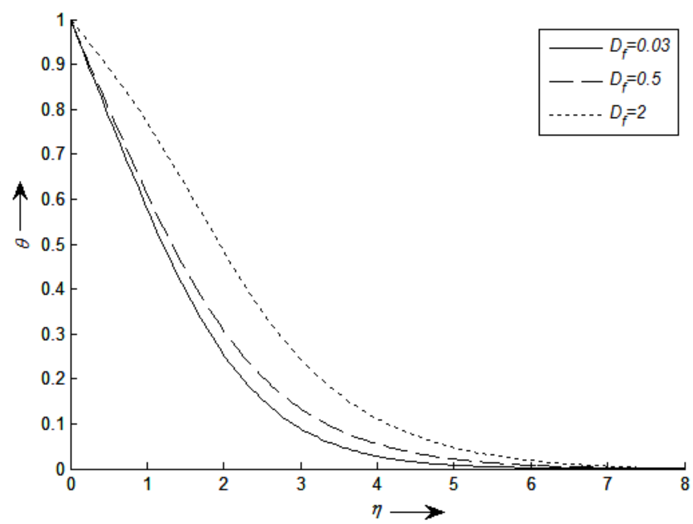

Fig.6: Temperature profile for different values of D_f

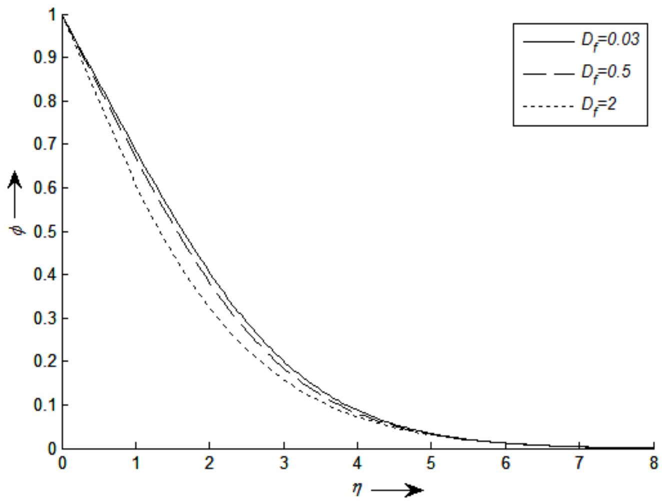

Fig.7: Concentration profile for different values of D_f

Figs.5, 6 and 7 show the Dufour effect on dimensionless horizontal velocity, temperature and concentration of species the fluid respectively for values of $N=1$, $Pr=0.72$, $Sc=0.57$, $S_T=0.6$.

Table:1

Pr	N	S_T	D_f	Sc	f''	$-\theta'$	$-\varphi'$
0.3	1	0.8	0.2	0.5	1.9709	0.2999	0.3711
0.7	1	0.8	0.2	0.5	1.7236	0.4266	0.3011
1.3	1	0.8	0.2	0.5	1.6043	0.5573	0.2414
0.7	1	1.6	0.2	0.5	1.8488	0.4494	0.2078
0.7	1	2	0.2	0.5	1.9113	0.4612	0.1559
0.7	1	0.6	0.0	0.5	1.6645	0.4327	0.3157
0.7	1	0.6	0.5	0.5	1.7358	0.3964	0.3340
0.7	1	0.6	2	0.5	1.9569	0.2171	0.4030
0.7	0	0.6	0.2	0.5	1.0182	0.3501	0.2678
0.7	0.5	0.6	0.2	0.5	1.3800	0.3914	0.3000
0.7	1.5	0.6	0.2	0.5	1.9612	0.4430	0.3401
0.7	1	0.6	0.2	0.2	1.9719	0.4619	0.2416
0.7	1	0.6	0.2	0.6	1.6779	0.4169	0.3285
0.7	1	0.6	0.2	1.2	1.5258	0.3872	0.4232

It has been noticed from the Fig.5 that horizontal velocity component of the fluid increases with the increase in the values of D_f. Fig.6 reveals that temperature of the fluid increases with increase in the values of D_f. It is true because large Dufour number causes high mass diffusivity and high concentration gradient which results in increase heat flux and hence temperature of the fluid rises. Fig.7 reveals that due to increase in the values of D_f concentration of species of the fluid decreases in the boundary layer.

The effect of the local skin friction, the local Nusselt number and the Sherwood number which are proportional to f'', $-\theta'$ and $-\varphi'$ which have practical importance are tabulated below in Table 1. The Table is self-explanatory.

CONCLUSION:

From the above discussions it is clear that the Soret and Dufour effects have a great rule to play in laminar flow, heat and mass transfer on a uniformly heated semi-infinite horizontal plate due to indirect natural convection. From the investigation it can be concluded that

- The effect of increase in the Soret number is to increase horizontal velocity component and concentration of species but to decrease temperature of the fluid in the boundary layer.
- The effect of increase in the Dufour number is to increase horizontal velocity component and temperature but to decrease concentration of species of the fluid in the boundary layer.

SYMBOLS USED:

Dimensional quantities

u- Fluid velocity components along x-axis
v- Fluid velocity components along y-axis,
p- Static pressure
p_∞-Working pressure
T- Temperature of the fluid in the boundary layer
C-Concentration of species in the boundary layer
C_w -Surface concentration,
C_∞ - Species concentration away from the plate
x - Dimensional length measured along the plate
y - Dimensional length measured normal to the plate

Greek Letters

α- Thermal diffusivity
υ -Kinematic viscosity
β_T, β_C -Volumetric coefficients of expansion of temperature and concentration of species
ρ -Fluid density
ψ - Dimensional Stream function
η - Similarity variable

ρ_∞ - Density of the quiescent fluid

Non-dimensional quantities

$\bar{u}$ - Fluid velocity components along *x*-axis
$\bar{v}$ - Fluid velocity components along *y*-axis,
g - Acceleration due to gravity,
l= Characteristic length
$\bar{p}$ - Static pressure
T^* -Temperature of the fluid in the boundary layer
D- Mass diffusivity,
C^* -Concentration of species in the boundary layer
$\bar{C}_w$ - Surface concentration,
$\bar{C}_\infty$ - Species concentration away from the plate
Pr- Prandtl number
N- Buoyancy parameter
S_T- Soret number
D_f - Dufour number
Sc-Schmidt number
Gr_L -Grashof number

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