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K-Contra Harmonic Mean Labeling of Some Graphs

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ABSTRACT

Let G be a (p, q) graph. A function f is called a k -contra harmonic mean labelling of a graph G if $f: V(G) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ in such a way that the function

$f^*: E(G) \rightarrow \{k, k+1, k+2, \dots, k+q-1\}$ defined as,

$f^*(e = uv) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$ or $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$ edge labels. The graph which admits k -contra harmonic mean labelling is called k -Contra harmonic mean graph.

KEYWORDS : k -Contra Harmonic mean labeling, K -Contra Harmonic mean graphs, Path, Cycle, Comb, etc.

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1. INTRODUCTION

By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary².

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J.A. Gallian (2016) can be found in¹. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

All graphs in this paper are simple, finite, undirected. Let G be a graph with p vertices and q edges. For a detail survey of graph labeling we refer to Gallian¹. For all other standard terminology and notation we follow Harary². S. Somasundaram and R. Ponraj introduced mean labeling for some standard graphs in 2013. S.S. Sandhya and S. Somasundaram introduced Harmonic mean labeling of graph. S. S. Sandhya, S. Somasundaram and J. Rajeshni Golda introduced Contra Harmonic mean labeling of graphs⁹.

We have introduced K- Contra Harmonic mean labeling. In this paper we investigate the k -Contra Harmonic mean labeling behaviour of some special graphs. The following definitions are useful for our present study.

Definition 1.1 Let G be a (p, q) graph. A function f is called a k -contra harmonic mean labelling of a graph G if $f : V(G) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ in such a way that the function $f^* : E(G) \rightarrow \{k, k+1, k+2, \dots, k+q-1\}$ defined as

$f^*(e = uv) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$ or $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$ with distinct edge labels. The graph which admits k -contra harmonic mean labeling is called k -contra harmonic mean graph.

Definition 1.2 The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$.

Definition 1.3 The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2 .

Definition 1.4 A Triangular ladder $TL_n, n \geq 2$ is a graph obtained from a ladder L_n by adding the edges $u_i v_{i+1}$, for $1 \leq i \leq n-1$ where u_i and v_i for $1 \leq i \leq n$, are the vertices of L_n . Such that $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ are two paths of length n in L_n .

Definition 1.5 An (m, n) kite graph consists of cycle of length m with n edges path attached to one vertex of a cycle.

Definition 1.6 Comb is a graph obtained by joining a single pendant edge to each vertex of a path.

2.MAIN RESULTS

Theorem 2.1. The path P_n is a k -contra harmonic mean graph for all k and $n \geq 2$.

Proof: Let $V(P_n) = \{v_i \mid 1 \leq i \leq n\}$ and $E(P_n) = \{e = v_i v_{i+1} \mid 1 \leq i \leq n-1\}$

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ by

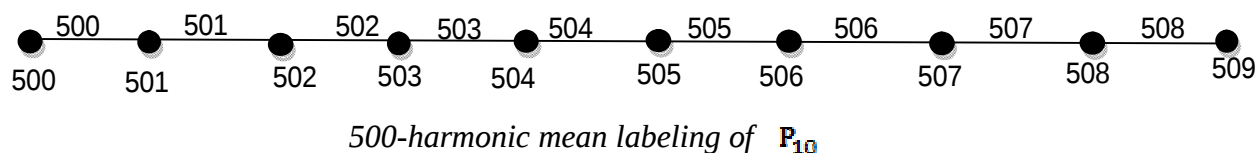
$$f(v_i) = k + i - 1, 1 \leq i \leq n$$

Then the induced edge labels are

$$f^*(e_i) = k + i - 1, 1 \leq i \leq n-1$$

The above defined function f provides k - contra harmonic mean labeling of the graph. Hence P_n is a k - contra harmonic mean graph.

Example 2.2



Theorem 2.3 The cycle graph C_n is a k -contra harmonic mean graph.

Proof: Let $u_1, u_2, \dots, u_n, u_1$ be the given cycle of length n .

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ by

$$f(u_i) = k + i - 1, \text{ for } 1 \leq i \leq n-1,$$

$$f(u_i) = k + q, \text{ for } i = n.$$

Then the induced edge labels are

$$f^*(u_i u_{i+1}) = k + i - 1, \text{ for } 1 \leq i \leq n-2$$

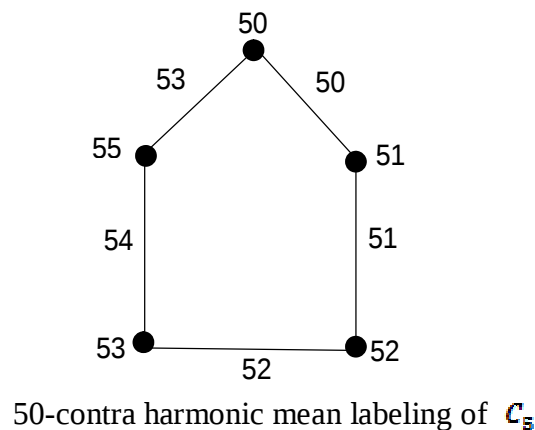
$$f^*(u_i u_{i+1}) = k + q - 1, \text{ for } i = n-1$$

$$f^*(u_1 u_n) = k + q - 2, \text{ for } i = n$$

The above defined function f provides k - contra harmonic mean labeling of the graph.

Hence C_n is a k - contra harmonic mean graph.

Example 2.3



Theorem 2.4 The Triangular ladder TL_n is k -contraharmonic mean graph for all k and $n \geq 2$.

Proof: Let $V(TL_n) = \{u_i, v_i \mid 1 \leq i \leq n\}$ and

$$E(TL_n) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1} \mid 1 \leq i \leq n-1\} \cup \{u_i v_i \mid 1 \leq i \leq n\}.$$

First we label the vertices as follows

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ by

$$f(u_i) = k + 4i - 3, \text{ for } 1 \leq i \leq n$$

$$f(v_1) = k$$

$$f(v_i) = k + 4i - 5, \text{ for } 2 \leq i \leq n$$

Then the induced edge labels are

$$f^*(u_i u_{i+1}) = k + 4i - 1, \text{ for } 1 \leq i \leq n-1$$

$$f^*(v_i v_{i+1}) = k + 4i - 3, \text{ for } 1 \leq i \leq n-1$$

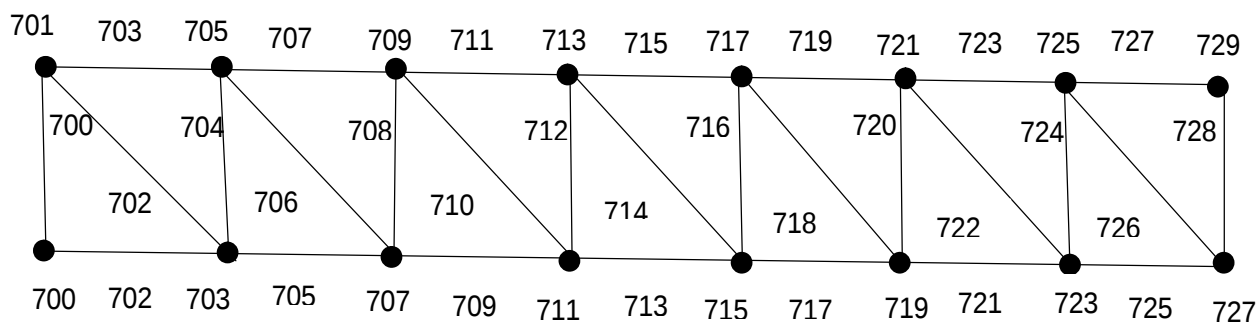
$$f^*(u_i v_i) = k + 4i - 4, \text{ for } 1 \leq i \leq n$$

$$f^*(u_i v_{i+1}) = k + 4i - 2, \text{ for } 1 \leq i \leq n-1$$

The above defined function f provides k -contraharmonic mean labeling of the graph.

Hence TL_n is a k -contraharmonic mean graph.

Example: 2.4



Theorem 2.5 A graph obtained by attaching a triangle at each pendent vertex of a comb is k - Contra harmonic mean graph for all k .

Proof: Let G be a graph obtained by attaching a triangle K_3 at each pendent vertex of $P_n \odot K_1$. Let u_i, v_i be the vertices of the comb $P_n \odot K_1$ in which v_i is joined with the vertex u_i of P_n . Let x_i, y_i, z_i be the vertices of i^{th} copy of K_3 . Identify z_i with $v_i, 1 \leq i \leq n$.

The resultant graph is G whose edge set is

$$E = \{u_i u_{i+1} \mid 1 \leq i \leq n-1\} \cup \{u_i v_i, v_i x_i, v_i y_i, x_i y_i \mid 1 \leq i \leq n\}.$$

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ by

$$f(u_i) = k + 5i - 3, \text{ for } 1 \leq i \leq n$$

$$f(v_i) = k + 5i - 2, \text{ for } 1 \leq i \leq n$$

$$f(x_i) = k + 5i - 5, \text{ for } 1 \leq i \leq n$$

$$f(y_i) = k + 5i - 4, \text{ for } 1 \leq i \leq n$$

Then the induced edge labels are

$$f^*(u_i u_{i+1}) = k + 5i - 1, \text{ for } 1 \leq i \leq n-1$$

$$f^*(u_i v_i) = k + 5i - 2, \text{ for } 1 \leq i \leq n$$

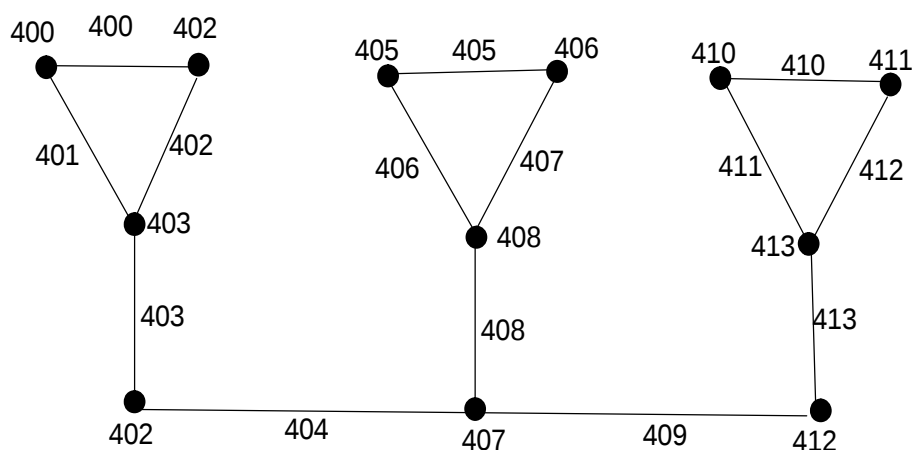
$$f^*(v_i x_i) = k + 5i - 4, \text{ for } 1 \leq i \leq n$$

$$f^*(v_i y_i) = k + 5i - 3, \text{ for } 1 \leq i \leq n$$

$$f^*(x_i y_i) = k + 5i - 5, \text{ for } 1 \leq i \leq n$$

The above defined function f provides k -contra harmonic mean labeling of the graph. Hence the graph G is k - contra harmonic mean graph.

Example: 2.6



400 -Contra harmonic mean labeling of G

Theorem 2.7 $P_n \odot K_1$ is k - contra harmonic mean labelling

Proof: Let $v_1, v_2, \dots, v_n$ be the path P_n . Let w_i be the vertices which is joined to the vertex $v_i, 1 \leq i \leq n$ of the path P_n . The resultant graph is $P_n \odot K_1$.

Let $G = P_n \odot K_1$. Define a function $f: V(G) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ by

$$f(v_i) = k + 2i - 2 \text{ for } 1 \leq i \leq n$$

$$f(w_i) = k + 2i - 1 \text{ for } 1 \leq i \leq n$$

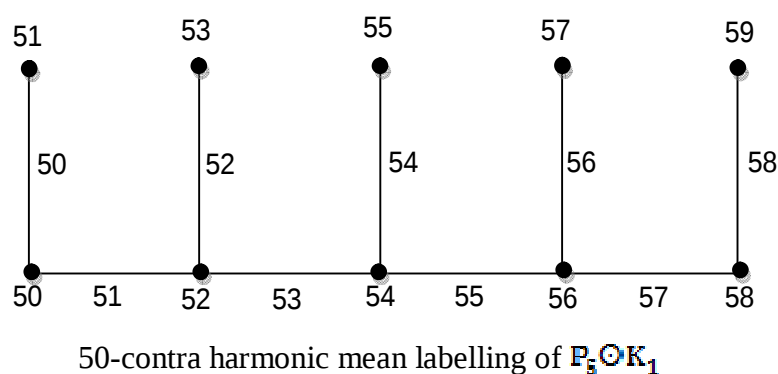
Then the distinct edge labels are

$$f^*(v_i v_{i+1}) = k + 2i - 1, \text{ for } 1 \leq i \leq n - 1$$

$$f^*(v_i w_i) = k + 2i - 2, \text{ for } 1 \leq i \leq n$$

The above defined function f provides k -contra harmonic mean labelling of the graph. Hence $P_n \odot K_1$ is k -contra harmonic mean labelling.

Example: 2.8



Theorem: 2.9A Triangular snake $T_n (n \geq 2)$ is k -contra harmonic mean graph $\forall k \geq 2$.

Proof: Let $V(T_n) = \{u_i | 1 \leq i \leq n\} \cup \{v_i | 1 \leq i \leq n-1\}$ and

$$E(T_n) = \{u_i u_{i+1}, u_{i+1} v_i, u_i v_i | 1 \leq i \leq n-1\}.$$

First we label the vertices as follows.

Define a function $f: V(T_n) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ by

$$f(u_i) = k + 3i - 3, \text{ for } 1 \leq i \leq n$$

$$f(v_1) = k + 1$$

$$f(v_i) = k + 3i - 2, \text{ for } 2 \leq i \leq n-1$$

Then the induced edge labels are

$$f^*(u_1 u_2) = k + 1$$

$$f^*(u_i u_{i+1}) = k + 3i + 1, \text{ for } 2 \leq i \leq n-1$$

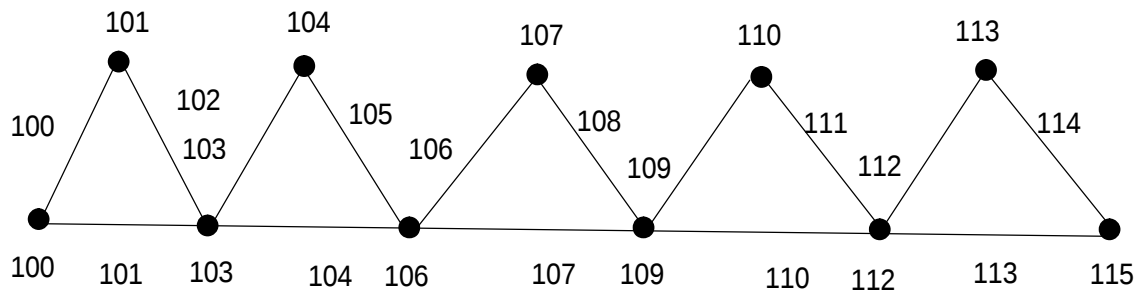
$$f^*(u_i v_i) = k + 3i - 3, \text{ for } 1 \leq i \leq n-1$$

$$f^*(u_{i+1} v_i) = k + 3i - 1, \text{ for } 2 \leq i \leq n-1$$

$$f^*(u_2 v_1) = k + 2$$

The above defined function f provides k -contra harmonic mean labeling of the graph. Hence T_n is a k - Contra harmonic mean graph.

Example 2.10



100- Contra harmonic mean graph of T_6

Theorem 2.11A (m, n) kite graph G is a k -contra harmonic mean graph.

Proof: Let $u_1, u_2, \dots, u_m, u_1$ be the given cycle of length m and $v_1, v_2, \dots, v_n$ be the given path of length n .

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ by

$$f(u_i) = k + i - 1, \text{ for } 1 \leq i \leq m,$$

$$f(v_i) = k + i + 5, \text{ for } 1 \leq i \leq n.$$

Then the induced edge labels are

$$f^*(u_i u_{i+1}) = k + i - 1, \text{ for } 1 \leq i \leq m - 1$$

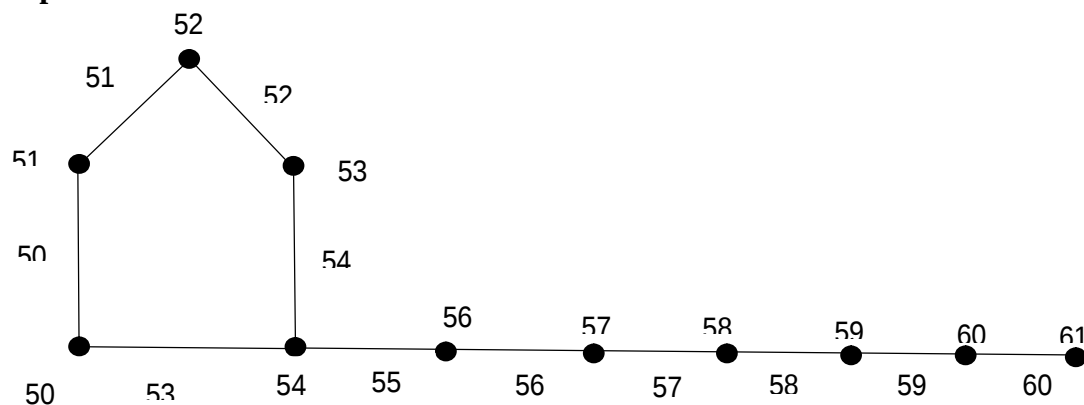
$$f^*(u_m u_{m-1}) = k + m - 1$$

$$f^*(u_1 u_m) = k + 3$$

and the edge labels of the path are $\{k + m + 1, k + m + 2, \dots, k + m + n - 1\}$. The above defined function f provides k -contra harmonic mean labeling of the graph.

Hence the (m, n) kite graph is a k -contra harmonic mean graph.

Example 2.12



50 -contra harmonic mean labeling of $(5,6)$ kite graph

Theorem 2.13 Let P_n be the path and G be the graph obtained from P_n by attaching C_3 in both the end edges of P_n . Then G is a k -contra harmonic mean graph.

Proof: Let P_n be the path $u_1, u_2, \dots, u_n$ and $v_1 u_1 u_2, v_2 u_{n-1} u_n$ be the triangles at the end.

Define a function $f : V(G) \rightarrow \{k, k+1, k+2, \dots, k+q\}$ by

$$f(u_i) = k + i, \text{ for } 1 \leq i \leq n,$$

$$f(v_1) = k; f(v_2) = k + q.$$

Then the induced edge labels are

$$f^*(u_i u_{i+1}) = k + i + 1, \text{ for } 1 \leq i \leq n - 1$$

$$f^*(u_1 v_1) = k$$

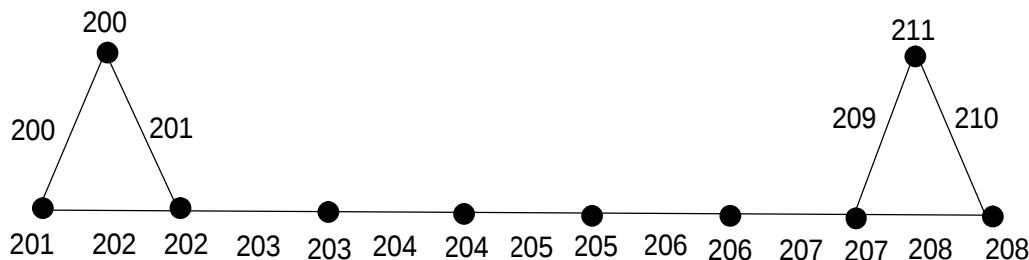
$$f^*(u_2 v_1) = k + 1$$

$$f^*(u_{n-1} v_2) = k + n + 1$$

$$f^*(u_n v_2) = k + n + 2$$

The above defined function f provides k -contra harmonic mean labelling of the graph. Hence G is a k -contra harmonic mean graph.

Example 2.14: A k -contra harmonic mean labelling of G obtained from P_8 is



200-contra harmonic mean labelling of G

3. CONCLUSION

The Study of labelled graph is important due to its diversified applications. It is very interesting to investigate graphs which admit k -Contra Harmonic Mean Labelling. In this paper, we proved that Path, Triangular Ladder TL_n , a graph obtained by attaching a triangle at each pendent vertex of a comb, Comb, Triangular Snake, (m, n) Kite graph, the graph obtained from P_n by attaching C_3 in both the end edges of P_n are k -Contra Harmonic Mean Graphs. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

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