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Study on Fuzzy Time Invariant Series Models for Crop Production Forecasting

Amit Kumar Rana

Assistant Professor, Department of Mathematics, Keral Verma Subharti College of Science, Swami Vivekanand Subharti University, Meerut- 250005, Uttar Pradesh, INDIA.
E-mail: rana2977@gmail.com

ABSTRACT

This paper presents the development of fuzzy time series; time invariant model development and its implementation in forecasting the agricultural production. It contains a comparative study of three models and their testing for the forecasting of Lahi crop production of University farm. The time series forecasting is based on the historical data of 21 years of University farm. The robustness of the models is examined on the basis of error estimates. The study reveals some interesting feature of fuzzy time series forecasting for the production of Lahi crop and can be used for short term forecasting of agriculture crop production.

KEY WORDS Fuzzy Time Invariant Series, Fuzzy Relations, Defuzzification.

***Corresponding author:**

Amit Kumar Rana

Assistant Professor, Department of Mathematics,
Keral Verma Subharti College of Science,
Swami Vivekanand Subharti University,
Meerut- 250005, Uttar Pradesh, INDIA.

E-mail: rana2977@gmail.com

I. INTRODUCTION

The fuzzy set theory has advanced in a number of ways and in many disciplines. Fuzzy set theory has applications in artificial intelligence, computer science, medicine, control engineering, decision theory, expert systems, logic, management science, operations research, pattern recognition, and robotics Fuzzy Set theory is an intellectual adventure in which the philosophy of Mathematics: abstraction and idealization are combined. In fuzzy sets and system, the process of abstraction and idealization makes possible the rigorous logical deductions to linguistic variables rather strictly to the crisp numbers. A time series is a sequence of observations taken sequentially in time with an intrinsic feature that the typically adjacent observations are dependent. The time series analysis is concerned with techniques for analysis of such dependence. Future values of time series data of agricultural production process are neither exactly governed by a mathematical function nor by a probability distribution. In such a process non-conventional techniques like fuzzy time series analysis be preferred as it uses the relation of dependency, a generalization of function.

Fuzzy set theory was introduced by Zadeh in 1965, after that a lot of work is done on fuzzy set theory and fuzzy logic. Mamdani[1] used fuzzy logic to approximate reasoning using linguistic synthesis .Duboi[2] used Fuzzy sets in approximate reasoning. Song and Chissom[3] presented the definition of fuzzy time series and outlined this modeling by means of fuzzy relation equations and approximate reasoning. Song and Chissom[4,5] applied the fuzzy time series models to forecast the enrolments of the University of Alabama using historical data. Chen[6] proposed an alternative simplified method of defuzzification using arithmetic operations. Wu[7] reveals some interesting features using fuzzy reasoning and fuzzy relational equations. Hwang[8] studied some forecasting problems using fuzzy time series. Huarng[9] proposed heuristic models of fuzzy time series for forecasting by the problem specific heuristic knowledge with Chen[6]'s model to improve the forecasting. Olmedo[10] studied the forecasting of Spanish unemployment using near neighbour and neural net techniques. De Oliveira[11] explained forecasting through distributed PSO-ARIMA-SVR hybrid system for time series forecasting. Javedani[12] analyses multilayer stock forecasting model using fuzzy time series. Egrioglu[13] have given interesting results using artificial neural network. Rubio[14] presented improving stock index forecasting by using a new weighted fuzzy-trend time series method. Ravi[15] fuzzy formal concept analysis based opinion mining for CRM in financial services. Che Ngoc[16] developed an improved fuzzy time series forecasting Model. Guo[17] describe the Fuzzy time series forecasting based on axiomatic fuzzy set theory. Pritpal[18] studied a hybrid fuzzy time series forecasting model based on granular computing and bio-inspired optimization approaches.

In the present work the time invariant model have been developed refined and applied on the forecasting of Lahi crop production. The objective of applying the fuzzy time invariant series forecasting models is to develop better forecasting models for the prediction of crop yield, a real non-deterministic process. Further, the area specific crop yield forecasting will help in better crop and agro based business planning and can be used in economics and business analysis.

II. FUZZY TIME SERIES

Let U universe of discourse; U={u_1, $u_{2,\dots}$ u_n}. A fuzzy set A on U is defined as

$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 \dots\dots\dots\dots\dots f_A(u_n)/u_n$ **(1)**

Where f_A is membership function of A, f:U→[0,1] and $f_A(u_i)$ indicates the grade of membership of u_i in A where $f_A(u_i) \in [0,1]$ and i= 1,2…n

Definition 2.1

Let U(t) (t=0,1,2…) a subset of R be universe of discourse on which fuzzy sets $f_i(t)$;(i=1,2,..) are defined and F(t) be the collection of $f_i(t)$,then F(t) is called fuzzy time series on U(t).

Definition 2.2

Suppose F (t) is caused by either F(t-1) only or by F(t-2) or F(t-3) or . . .or f(t - m) m>0. This relation can be expressed as the following fuzzy relational equation

$F(t)=F(t-1)oR(t,t-1)$ **(2)**

Or

$F(t) = (F(t-1) \cup F(t-2) \cup \dots\dots \cup F(t-m))oR_0(t,t-m)$ **(3)**

And is called first order model of F(t).

Definition 2.3

Suppose F (t) is caused by either F(t-1) ,F(t-2) ,F(t-3) . . .and f(t - m) m>0 simultaneously. This relation can be expressed as the following fuzzy relational equation

$F(t) = (F(t-1) \times F(t-2 \times \dots F(t-m))oR_{a,}(t,t-m)$ **(4)**

And is called m^{th} order model of F(t).

Definition 2.4

If in (2) or (3) or (4) the fuzzy relation R(t,t-1), $R_0(t,t-m)$ or $R_{a,}(t,t-m)$ of F(t) is independent of time t that is for different times t_1 and $t_{2,}$

$R(t_1,t_1-1)= R(t_2,t_2-1)$, or$R_0(t_1,t_1-1)= R_0(t_2,t_2-1)$ or $R_a(t_1,t_1-1)= R_a(t_2,t_2-1)$, then F(t) is called a time invariant fuzzy time series otherwise it is called a time variant fuzzy time series.

III. FIRST ORDER FUZZY RELATIONS IN TIME INVARIANT MODEL

Suppose F (t) as a fuzzy time series (t= 0, 1,2, ...). For the first order model R (t, t-1) of F(t) for any $f_i(t) \in F(t)$ where $j \in J$ there exist an $f_i(t-1) \in F(t-1)$ where $i \in I$ then the fuzzy relation R_{ij} (t, t-1) is such that

$$f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1) \quad \textbf{(5)}$$

which is equivalent to IF ….THEN rule as "IF f_i (t-1) THEN f_j (t)",

$$R_i(t, t-1)=\cup_{ij} R_{ij}(t, t-1) \quad \textbf{(6)}$$

$$R_{ij}(t, t-1)= f_i(t-1)\times f_i(t-1) \quad \textbf{(7)}$$

Here the operator “o” is called Mamdami type max min operator.

IV. FUZZY TIME SERIES PRODUCTION FORECASTING: *ALGORITHM*

The proposed fuzzy time series forecasting model and its implementation uses the following steps:

Step 1 Define the universe of discourse with given time series data on which fuzzy sets are to be defined.

Step 2 Partitioning the universe of discourse into several even length intervals.

Step 3 Define the fuzzy sets (linguistic variables) on universe of discourse.

Step 4 Fuzzification of time series data for fuzzy input.

Step 5 Computing the fuzzy relationships.

Step 6 Computing the forecasted production (fuzzy output).

Step 7 Defuzzification of fuzzy output for crisp forecasting.

V. COMPUTATIONAL PROCEDURES

The implementation of the above algorithm for the production forecasting of the Lahi crop is based on the 21 years (1996-97 to 2015-2016) time series production data of the University farm

Step 1. Define the universe of discourse to accommodate the time series data. It needs the minimum and maximum production and set as D_{min} and D_{max}. Thus universe of discourse U is defined as [D_{min} –D_1, D_{max} - D_2] here D_1 and D_2 are two proper positive numbers. In the present case of production forecasting universe of discourse is U=[400-1100]

Step 2. Partition the universes of discourse into 7 equal length intervals $u_1, u_2,...., u_7$ such that

u, = [400-500],

u_2 = [500-600],

u_3 = [600-700]

u_4 = [700-800],

u_5 = [800-900],

u_6 = [900-1000],

u_7= [1000-1100]

Step 3. Define 7 fuzzy sets A_1, A_2.......A_7 having some linguistic values on the universe of discourse U. The linguistic values to this fuzzy variable are:

A_1	:	poor production
A_2	:	below average production
A_3	:	average production
A_4	:	good production
A_5	:	very good production
A_6	:	excellent production
A_7	:	bumper production

These fuzzy sets in terms of its membership to different intervals are expressed as follows:

A_1	:	$[1/u_1 , .5/u_2, 0/u_3, 0/u_4, 0/u_5 , 0/u_6, 0/u_7]$
A_2	:	$[.5/u_1, 1/u_2, 5/u_3, 0/u_4, 0/u_5 ,0/u_6, 0/u_7]$
A_3	:	$[0/u_1, .5/u_2, l/u_3, .5/u_4, 0/u_5 , 0/u_6, 0/u_7]$
A_4	:	$[0/u_1, 0/u_2, .5/u_3, 1/u_4, .5/u_5 ,0/u_6, 0/u_7]$
A_5	:	$[0/u_1, 0/u_2, 0/u_3, .5/u4, l/u_5, .5/u_6 ,0/u_7]$
A_6	:	$[0/u_1, 0/u_2, 0/u_3 , 0/u_4 , .5/u_5 , 1/u_6, .5/u_7]$
A_7	:	$[0/u_1, 0/u_2, 0/u_3, 0/u_4, 0/u_5, .5/u_6, 1/u_7]$

Step 4. Fuzzification of the time series data for the fuzzy input to the models are obtained as:

Table 1 Fuzification of time series Lahi production data

Year	Production Kg/hect	A_1	A_2	A_3	A_4	A_5	A_6	A_7	Fuzzified production
96-97	1025	0	0	0	0	.2	.7	1	A_7
97-98	512	.9	1	.5	0	0	0	0	A_2
98-99	1005	0	0	0	0	.5	.95	1	A_7
99-00	852	0	0	0	.5	1	.7	.2	A_5
00-01	440	1	.5	0	0	0	0	0	A_1
01-02	502	.95	1	.5	0	0	0	0	A_2
02-03	775	0	0	.5	1	.9	.4	0	A_4
03-04	465	1	.8	.3	0	0	0	0	A_1
04-05	795	0	0	.5	1	.9	.4	0	A_4
05-06	970	.0	.0	0	0	.5	1	.8	A_6
06-07	742	0	.2	.7	1	.5	0	0	A_4
07-08	635	.3	.8	1	.7	.2	0	0	A_3
08-09	994	0	0	0	0	.5	1	.9	A_6
09-10	759	0	0	.5	1	.7	.2	0	A_4
10-11	883	0	0	0	.5	1	.8	.3	A_5
11-12	599	.5	1	.99	.5	0	0	0	A_1
12-13	499	1	.5	0	0	0	0	0	A_1
13-14	590	.5	1	.8	.3	0	0	0	A_2
14-15	911	0	0	0	.3	.8	1	.7	A_6
15-16	862	0	0	0	.5	1	.8	.5	A_5
16-17	801								

Step 5. The fuzzy logical relations have obtained from the table 1 are as:

Table 2. Fuzzy logical relationships of the historical lahi production.

1	$A_1 \longrightarrow A_2$	$A_1 \longrightarrow A_2$	$A_1 \longrightarrow A_4$	$A_2 \longrightarrow A_1$	$A_2 \longrightarrow A_4$
2	$A_2 \longrightarrow A_6$	$A_2 \longrightarrow A_7$	$A_3 \longrightarrow A_6$	$A_4 \longrightarrow A_1$	$A_4 \longrightarrow A_3$
3	$A_4 \longrightarrow A_5$	$A_4 \longrightarrow A_6$	$A_5 \longrightarrow A_1$	$A_5 \longrightarrow A_2$	$A_6 \longrightarrow A_7$
4	$A_6 \longrightarrow A_4$	$A_6 \longrightarrow A_5$	$A_7 \longrightarrow A_2$	$A_7 \longrightarrow A_5$	

Further the fuzzy logical relations obtained in table 2 are arranged such that consider the relations only once , i.e. leaving the repeated relations, we thus obtain a total of 17 logical relations the are placed in table 3.

Table 3. Fuzzy logical relationship groups

1	$A_1 \longrightarrow A_2$	$A_1 \longrightarrow A_4$		
2	$A_2 \longrightarrow A_1$	$A_2 \longrightarrow A_4$	$A_2 \longrightarrow A_6$	$A_2 \longrightarrow A_7$
3	$A_3 \longrightarrow A_6$			
4	$A_4 \longrightarrow A_1$	$A_4 \longrightarrow A_3$	$A_4 \longrightarrow A_5$	$A_4 \longrightarrow A_6$
5	$A_5 \longrightarrow A_1$	$A_5 \longrightarrow A_2$		
6	$A_6 \longrightarrow A_4$	$A_6 \longrightarrow A_5$		
7	$A_7 \longrightarrow A_2$	$A_7 \longrightarrow A_5$		

Thus with the fuzzy logical relations in Table 9 a total of 17 fuzzy relations can be obtained. The fuzzy time invariant relation R can be obtained as:

$R = ¿ i = 1 ¿ 17\, R_i$, *here is the* $\cup$ *operator*.

Thus computing all the fuzzy logical relations R_1 ,....,R_{15} and Taking the union of these computed relations we obtain a fuzzy time invariant relation R as :

$$R = \begin{pmatrix} 5 & 1 & 1 & .5 & .5 & 1 & .5 \\ 1 & .5 & .5 & 1 & .5 & 1 & 1 \\ .5 & .5 & .5 & .5 & .5 & 1 & .5 \\ 1 & .5 & 1 & .5 & 1 & 1 & .5 \\ 1 & 1 & .5 & .5 & .5 & .5 & .5 \\ .5 & .5 & .5 & 1 & 1 & .5 & 0 \\ .5 & 1 & .5 & .5 & 1 & .5 & 0 \end{pmatrix}$$

Step 6. Computation of fuzzy forecast of the crop production have been carried by the three models: Chen's arithmetic model (Model-1), Srivastava and Singh's refined arithmetic model (Model-2) and Song and Chissom model (Model-3)

Model-1

1. If the fuzzified production of the year i is A_j , and there is only one fuzzy logical relationship in the fuzzy logical relationship groups in Table 3 in which the current state of the production is A_j , then the fuzzy forecasted production of the year i+1 is A_j.
2. If the fuzzified production of the year i is Aj and there are fuzzy logical relationships in the fuzzy logical relationship group as:

$A_j \longrightarrow A_{k1}$,$A_j \longrightarrow A_{k\,2}$,….. $A_j \longrightarrow A_{kp}$, the A_j forming a relation with A_{k1}, A_{k2},....., A_{kp} is the fuzzy forecasted production for the year i+1

Model-2

Similar to model-1 but it also counts the repeated relations and the frequency is recorded and is used for defuzzification (crisp output).

Model-3

It uses the fuzzy time invariant relation R computed in sep 5. If A_{i-1} is the production of the year i-1, the fuzzy forecasted production for the year i will be A_i and will be computed by $A_i = A_{i-1}$ o R. The computed fuzzy output is as

Table 4. Fuzzy output of the models

Year	A_1	A_2	A_3	A_4	A_5	A_6	A_7
96-97	.5	1	.5	.7	1	.5	.2
97-98	1	.9	.5	1	.9	1	1
98-99	.5	1	.5	.95	1	.5	.5
99-00	1	1	.5	.7	.7	.5	.5
00-01	.5	1	.5	.5	1	.5	.5
01-02	1	.95	.5	1	.95	1	1
02-03	1	.9	1	.5	1	1	.5
03-04	.8	1	.5	.8	1	.8	.8
04-05	1	.9	1	.5	1	1	.5
05-06	.5	.8	.5	1	1	.5	.5
06-07	1	.5	1	.5	1	1	.5
07-08	.8	.5	.7	.8	.7	1	.8
08-09	.5	.9	.5	1	1	.5	.5
09-10	1	.7	1	.5	1	1	.5
10-11	1	1	.5	.8	.8	.5	.5
11-12	1	.5	.5	1	.5	1	1
12-13	.5	1	.5	.5	1	.5	.5
13-14	1	.5	.5	1	.5	1	1
14-15	.8	.8	.5	1	1	.5	.5
15-16	1	1	.5	.8	.8	.5	.5

Step 7. Defuzzification is the process by which fuzzy output of model is transformed to crisp values for getting the forecasted values. The output of each of the three models are defuzzified in the following ways:

Model-1

1. If the production of the year i is A_j and fuzzy logical relation is $A_j \longrightarrow A_k$ and A_k has max membership in interval u_k, then the forecasted production for the year i=1 will be midpoint of A_k.
2. If the fuzzified production of the year i is A_j and there are fuzzy logical relationships in the fuzzy logical relationship group as:$A_j \longrightarrow A_{k1}$,$A_j \longrightarrow A_{k2}$,….. $A_j \longrightarrow A_{kp}$

A_{k1} , A_{k2} …… , A_{kp} has max membership in the intervals u_{k1},u_{k2},..., u_{kp} respectively and m_1 , m_2 ,....m_p are their respective midpoints , then the forecasted production for the year i+1 will be (m_1 + m_2...+m_p)/p.

3. If the fuzzified production of a year is A_j, and no logical relationship is found in logical relationship groups, whose current state of production is A_j,where the maximum membership value of A_j occurs at interval $u_j$and the midpoint of u_j. is$m_j$then the forecasted production of year $i + 1$ is m_j.

Model-2

Similar procedures of defuzzification as in model-1 with additional concept of repeated relations and according weighted mean is computed keeping in view of their frequencies.

Model-3

A combined approach is utilized to have the defuzzification of the fuzzy output in table 4 into crisp output. The rules are as follows;

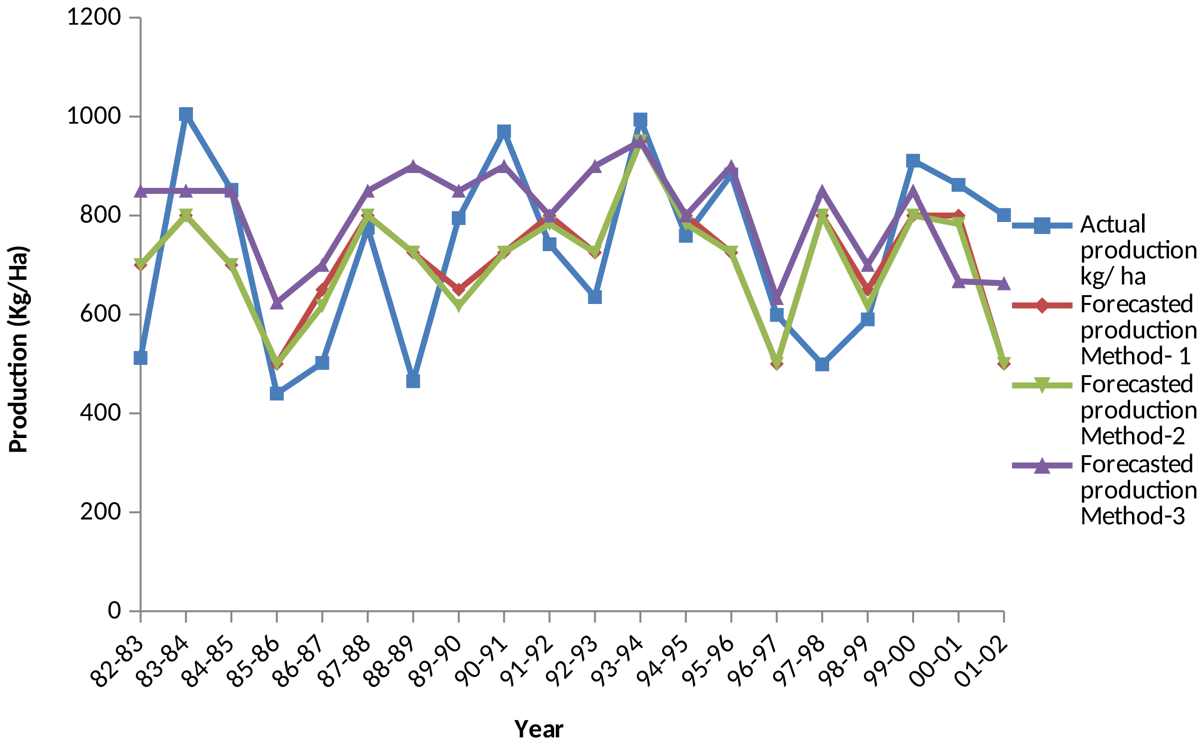
1. If fuzzy output set has only one max, then choose the midpoint of the interval corresponding to that max as forecasted production.
2. If the fuzzy output has only one consecutive max, then choose the midpoint of the corresponding conjunct interval as the forecasted value.
3. Otherwise compute the forecasted value by center of area (COA) or Centroid (COG) method.

VI. Computational Results

The forecasted production of Lahi obtained by the above three models have been place in the following table and is presented in fig.1.

Table 5. Lahi production forecast

Year	Actual production kg/ ha	Forecasted production Method- 1	Forecasted Production Method-2	Forecasted production Method-3
82-83	512	700	700	850
83-84	1005	800	800	850
84-85	852	700	700	850
85-86	440	500	500	624
86-87	502	650	617	700
87-88	775	800	800	850
88-89	465	725	725	900
89-90	795	650	617	850
90-91	970	725	725	900
91-92	742	800	783	800
92-93	635	725	725	900
93-94	994	950	950	950
94-95	759	800	783	800
95-96	883	725	725	900
96-97	599	500	500	633
97-98	499	800	800	850
98-99	590	650	617	700
99-00	911	800	800	850
00-01	862	800	783	667
01-02	801	500	500	663



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g. 1. Actual and forecasted Lahi production

VII. CONCLUSION

Three fuzzy time series forecasting models applied to this study, the results are almost similar as the average forecasting error of the three models is about 11%. Considering the fact, that agricultural production depends on various factors like weather or disease attack is a non-linear and complex phenomenon for which a probability or trend factor is difficult to locate. But most of work done in the area of crop production forecasting deals with the stochastic or probabilistic models, which necessarily need some probability distribution or trend in the time series data.

The observation of fuzzy output of the model-3 needs to be tackled by some index based defuzzification procedure for better crisp forecast. The model-3 clearly indicates that defuzzification methods like COA, COG or MOM methods are not suitable for present problem. The interesting features of fuzzy output of the model-3 can be harvested by using some higher order defuzzification method on introducing some indicators. This feature of model-3 gives an advantageous basis in comparison to other two models. The result of this study can provide a better basis to Farm administration for post harvest management of the crop yield and the local industries in planning for their raw material requirement management. The agri-business management can optimally utilize fuzzy time series forecasting.

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