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Iso-S-ClosednessandIso-S*-Closednessin L-Fuzzy Topological Spaces

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ABSTRACT

Alongtheline of is compactnessin L-fuzzy topological spaces, we introduce iso-S-closeness and iso-S*-closeness for arbitrary L-fuzzy subsets. Further CL-iso-S-closed and CL- iso-S*-closed L-fuzzy spaces are defined and studied some of the properties and obtain some relations of these spaces with other spaces.

KEYWORDS:L-fuzzyiscompactness, L-fuzzy CL- iso-S-closedness, L-fuzzy CL- iso-S*-closedness.

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INTRODUCTION

In [0,1] fuzzy topological space, S-closedness and S*-closedness were defined by Coker[10] and Malakar[15] , but the definitions are not studied in arbitrary fuzzy sets. Later Kudri and Warner [13] and Kudri[14] have introduced good definitions of S-closedness and S*-closedness in L-fuzzy topological spaces where L is a fuzzy lattice and have studied some of their properties along the line of compactness[11] . In 1970, Bacon [3] introduced the notion of isocompactness in general topology.Bhaumik and Bhattacharya [5] introduced isocompactness in L-fuzzy topological spaces, in which every L-fuzzy closed,countably compact subspaces are L-fuzzy compact. In this paper, using the concepts of S-closedness and S*-closedness in L-fuzzy topological spaces we introduce two new concepts namely iso-S-closedness and iso- S*- closedness for arbitrary L-fuzzy subsets and study some properties of these spaces. Further we generalize these concepts as CL-iso-S-closedness and CL-iso-S*-closedness in L-fuzzy topological spaces which are the stronger form of iso-S-closedness and iso- S*- closedness.

PRILIMINARIES

Throughout this paper X and Y will be non-empty ordinary sets and $L = L(\leq, \vee, \wedge, ')$ will denote a fuzzy lattice, i.e. a completely distributive lattice with a smallest element 0 and a largest element 1 ($0 \neq 1$), and with and order reversing involution $a \rightarrow a'$ ($a \in L$). An L-fuzzy subset on X is a mapping $\lambda : X \rightarrow L$, and the family of L-fuzzy subsets on X is denoted by L^X.X is called the carrier domain of each L-fuzzy subset on X.

Definition 2.1 An element p of L is called prime[1] if and only if $p \neq 1$ and whenever a, b $\in$ L with $a \wedge b \leq p$ then $a \leq p$ or $b \leq p$. The set of all prime elements of L will be denoted by pr (L).

Definition 2.2 An element α of L is called union-irreducible or coprime[1] if and only if whenever a, b$\in$ L with $\alpha \leq a \vee b$ then $\alpha \leq a$ or $\alpha \leq b$. The set of all nonzero union-irreducible elements of L will be denoted by M(L). It is obvious that $p \in pr(L)$ if and only if $p' \in M(L)$.

Definition 2.3Let (X, τ) be an L-fuzzy topological space and let $\lambda \in L^X$. The L-fuzzy set λ is called

i) Semiopen[2] if and only if there exists $\beta \in \tau$ such that $\beta \leq \lambda \leq cl(\beta)$ and semiclosed[2] if and. only if there exists a closed L-fuzzy set β such that $int(\beta) \leq \lambda \leq \beta$ that is λ' is semiopen

ii) Pre-open[13] if and only if $\lambda \leq int(cl(\lambda))$ and pre-closed[13] if and only if $cl(int(\lambda)) \leq \lambda$ that is λ' is pre-open.

iii) Regularly open[2] if and only if $\lambda = int(cl(\lambda))$ and λ is regularly closed[2] if and only if λ'

is regularly open i.e., $\lambda = cl(int(\lambda))$.

iv) Regularly semiopen[2] if and only if there exist a regularly open L-fuzzy set β such that$\beta \leq \lambda \leq cl(\beta)$ and λ is regularly semiclosed[2] if and only if λ' is regularly semiopen.

Definition 2.4Let (X, τ) and (Y, τ') be two L-fuzzy topological spaces. A function $f : (X, \tau) \rightarrow (Y, \tau')$ is called

i) Almost continuous[2] if and only if $f^{-1}(\lambda) \in \tau$ for all regularly open λ in (Y, τ').

ii) Almost open[13] if and only if $f(\lambda) \in \tau'$ for every regularly open λ in (X, τ).

iii) Weakly continuous[2] if and only if $f^{-1}(\lambda) \leq int(f^{-1}(cl(\lambda)))$ for all $\lambda \in \tau'$.

iv) Semi-weakly continuous[10] if and only if $f^{-1}(\lambda) \leq int{*}(f^{-1}(cl{*}(\lambda)))$ for all semiopen $\lambda \in \tau'$.

v) Irresolute[10] if and only if $f^{-1}(\lambda)$ is semi-open in (X, τ) for every semi-open L-fuzzy set λ in (Y, τ').

vi) Semi-irresolute[15] if and only if $f^{-1}(\lambda)$ is semiclopen in (X, τ) for every semiclopen L-fuzzy set λ in (Y, τ').

vii) Perfect[17] if and only if f is L-fuzzy continuous, L-fuzzy closed and for each $y \in Y$, $f^{-1}(y)$ is compact L-fuzzy subset in (X, τ).

Definition 2.5Let (X, τ) be an L-fuzzy topological space and $\lambda \in L^X$. Then

i) The L-fuzzy subset λ is said to be compact[12] if and only if for every $p \in Pr(L)$ and every collection $(\gamma_i)_{i \in J}$ of open L-fuzzy subsets with $(\vee_{i \in J}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$, there exits a finite subset F of J with $(\vee_{i \in F}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$.

If λ is the whole space, then we say that the L-fuzzy topological space (X, τ) is compact.

ii) The L-fuzzy subset λ is said to be semicompact[14] if and only if for every $p \in Pr(L)$ and every collection $(\gamma_i)_{i \in J}$ of semiopen L-fuzzy subsets with $(\vee_{i \in J}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$, there exits a finite subset F of J with $(\vee_{i \in F}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$.

If λ is the whole space, then we say that the L-fuzzy topological space (X, τ) is semicompact.

iii) The L-fuzzy subset λ is said to be S-closed[13] if and only if for every $p \in Pr(L)$ and every collection $(\gamma_i)_{i \in J}$ of semiopen L-fuzzy subsets with $(\vee_{i \in J}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$, there exits a finite subset F of J with $(\vee_{i \in F}\, cl\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$.

If λ is the whole space, then we say that the L-fuzzy topological space (X, τ) is S-closed.

iv) The L-fuzzy subset λ is said to be S∗-closed[14] if and only if for every $p \in Pr(L)$ and every collection $(\gamma_i)_{i\in J}$ of semi-open L-fuzzy subsets with $(\vee_{i\in J}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$, there exits a finite subset F of J with $(\vee_{i\in F}\, cl*(\gamma_i))(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$.

If λ is the whole space, then we say that the L-fuzzy topological space (X, τ) is S∗-closed.

Other characterizations of S-closedness and S∗-closed are given in Th.2.6, Th. 2.7 and Th. 2.8.

Theorem 2.6Let (X, τ) be an L-fuzzy topological space and $\lambda \in L^X$. The L-fuzzy subset λ is S-closed[13] if and only if for every $p \in Pr(L)$ and every collection $(\gamma_i)_{i\in J}$ of regularly closed L-fuzzy sets with $(\vee_{i\in J}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$, there is a finite subset F of J with $(\vee_{i\in F}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$.

Theorem 2.7Let (X, τ) be an L-fuzzy topological space and $\lambda \in L^X$. The L-fuzzy subset λ is S-closed[13] if and only if for every $p \in Pr(L)$ and every collection $(\gamma_i)_{i\in J}$ of regularly semiopen L-fuzzy sets with $(\vee_{i\in J}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$, there is a finite subset F of J with $(\vee_{i\in F}\, cl\, \gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$.

Theorem 2.8]Let (X, τ) be an L-fuzzy topological space and $\lambda \in L^X$. The L-fuzzy subset λ is S∗-closed[14] if and only if for every $p \in Pr(L)$ and every collection $(\gamma_i)_{i\in J}$ of semiclopen L-fuzzy sets with $(\vee_{i\in J}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$, there is a finite subset F of J with $(\vee_{i\in F}\gamma_i)(x) \nleq p$ for all $x \in X$ with $\lambda(x) \geq p'$.

Definition 2.9 Let (X, τ) be an L-fuzzy topological space and $\lambda \in L^X$. The L-fuzzy subset λ is said to be L-fuzzy isocompact[5] if every countably compact and closed L- fuzzy subset of λ is L-fuzzy compact.

If λ is the whole space, then L-fuzzy topological space (X, τ) is isocompact.

Definition 2.10 Let (X, τ) be an L-fuzzy topological space and $\lambda \in L^X$. The L-fuzzy subset λ is said to be semi-isocompact[8] if and only if every countably compact and closed L-fuzzy subset of λ is semi compact. If λ is the whole space, then the L-fuzzy topological space (X, τ) is also semi-is compact.

Theorem 2.11[13]Let (X, τ) be an S-closed L-fuzzy topological space. Then each regularly open L-fuzzy subset in (X, τ) is S-closed.

Theorem 2.12[13] Let (X, τ) and (Y, τ´) be L-fuzzy topological spaces and let f : (X, τ) →(Y, τ´) be an almost continuous, almost open mapping and let λ be an S-closed L-fuzzy subset of (X, τ). Then f (λ) is an S-closed L-fuzzy subset of (Y, τ´).

Proposition 2.13[14] Let (X, τ) and (Y, τ´) be L-fuzzy topological spaces and let f : (X, τ) →(Y, τ´) be a semi-irresolute mapping with $f^{-1}(y)$ is finite for every y ∈Y. If λ∈ L^X is S∗-closed in(X, τ), then f (λ) is S∗-closed in (Y, τ´).

Proposition 2.14[14] Let (X, τ) and (Y, τ´) be L-fuzzy topological spaces and let f : (X, τ) →(Y, τ´) be an irresolute mapping with $f^{-1}(y)$ is finite for every y ∈Y. If λ∈ L^X is S∗-closed in(X, τ), then f (λ) is S∗-closed in (Y, τ´).

Proposition 2.15[14] Let (X, τ) and (Y, τ´) be L-fuzzy topological spaces and let f : (X, τ) → (Y, τ´) be a semi-irresolute mapping with $f^{-1}(y)$ is finite for every y ∈Y. If λ∈ L^X is semicompactin (X, τ), then f (λ) is S∗-closed in(Y, τ´).

Proposition 2.16[14] Let (X, τ) and (Y, τ´) be L-fuzzy topological spaces and let f : (X, τ) →(Y, τ´) be a semiweekly continuous mapping with $f^{-1}(y)$ is finite for every y ∈Y. If λ∈ L^X is semicompactin (X, τ), then f (λ) is S∗-closed in (Y, τ´).

Theorem 2.17[2] If λ is an L-fuzzy subset of (X, τ), μ is an L-fuzzy subset of (Y, τ′) and X is product related to Y, then

a) Cl (λ x μ) = Clλ x Clμ and
b) Int (λx μ) = Intλ x Intμ hold.

Definition 2.18 An L-fuzzy topological space (X, τ) is called fully stratified[17] if for each p ∈L, the L-fuzzy set which takes constant value p at each point x ∈ X belongs to τ.

Theorem 2.19[17] If (X, τ) be a compact L-fuzzy topological space and (Y, τ′) be a fully stratified L-fuzzy topological space, then the projection mapping P_Y: X × Y→ Y is L-fuzzy perfect.

ISO-S-CLOSEDNESS AND ISO-S∗-CLOSEDNESS IN L-FUZZY TOPOLOGICAL SPACES

Definition 3.1 Let (X, τ) be an L-fuzzy topological space and λ ∈ L^X. The L-fuzzy subset λ is said to be iso-S-closed if and only if every closed countably compact subset of λ is S-closed.

If λ is the whole space, then the L-fuzzy topological space (X, τ) is also iso-S-closed.

Theorem 3.2

Let (X, τ) be an L-fuzzy topological space and $\lambda \in L^X$. Then the L-fuzzy subset λ is iso-S-closed if and only if every regular closed countably compact subset of λ is S-closed.

Proof: Since each regular closed set is closed then the result follows immediately from the definition 3.1.

Theorem 3.3

If an L- fuzzy topological space (X, τ) is the union of a countable collection of closed and iso-S-closed L-fuzzy subsets, then (X, τ) is L-fuzzy iso-S-closed.

Proof: Suppose $X = \vee\mu_i$, where each μ_i is closed and iso-S-closed L-fuzzy subset of X and let β be a closed and countably compact L-fuzzy subset of X. Let $p \in pr(L)$ and let $\{\gamma_i\}_{i\in J}$ be a family of semi-open L-fuzzy sets with $(\vee_{i\in J}(\gamma_i))(x) \nleq p$ for all $x \in X$ such that $\beta(x) \geq p'$. For each i, $\beta\wedge\mu_i$ is a closed, countably compact L-fuzzy subset of μ_i. So it is S-closed L-fuzzy subset, since each μ_i is L-fuzzy iso-S-closed. By S-closedness of $\beta\wedge\mu_i$, there exist a finite subset F of J with $(\vee_{i\in F}(cl\gamma_i))(x) \nleq p$ for all $x\in X$ such that $(\beta\wedge\mu_i)(x) \geq p'$ i.e. $\beta(x) \geq p'$. Hence β is a S-closed L-fuzzy subset, which implies that X is L-fuzzy iso-S-closed.

Theorem 3.4

Let $f:(X, \tau) \rightarrow (Y, \tau')$ be an L-fuzzy perfect, almost continuous and almost open mapping from an iso-S-closed L-fuzzy topological space (X, τ) onto an L- fuzzy topological space (Y, τ'). Then (Y, τ') is L-fuzzy iso-S-closed.

Proof : Let β be a regular closed and countably compact L-fuzzy subset of (Y, τ'). Since f is L-fuzzy perfect map, then $f^{-1}(\beta)$ is closed and countably compact L-fuzzy subset of (X, τ). By L-fuzzy iso-S-closednessof (X, τ), $f^{-1}(\beta)$ is L-fuzzy S-closed. Since f is onto L-fuzzy almost continuous and almost open mapping then $ff^{-1}(\beta) = \beta$ is S-closed [by 2.12] L-fuzzy subset in (Y, τ'). Hence (Y, τ') is L-fuzzy iso-S-closed.

Theorem 3.5

If (X, τ) and (Y, τ') be two S-closed L-fuzzy topological spaces such that X is product related to Y, then $X \times Y$ is L-fuzzy S-closed.

Proof: Let $\{\lambda_i\times\beta_i : i\in I\}$ be an L-fuzzy cover of $X \times Y$ by semi-open L-fuzzy sets of $X \times Y$, where λ_i's and β_i's are semi-open L-fuzzy sets in X and Y respectively. Then $\{\lambda_i : i\in I\}$ and $\{\beta_i : i\in I\}$ are L-fuzzy semi-open covers of X and Y respectively. As (X, τ) and (Y, τ') are S-closed L-fuzzy topological spaces then there exist finite subsets M and N of I such that, $(\vee_{i\in M}(cl\lambda_i))(x) \nleq p$ and $(\vee_{i\in N}(cl\beta_i))(x) \nleq p$.

Now, $\{\vee\, cl(\lambda_i\times\beta_i): i\in M \vee N\}(x) = [\vee\{cl\,\lambda_i: i\in M \vee N\}](x) \times [\vee\{cl\,\beta_i : i\in M \vee N\}](x) \nleq p$

Hence the proof.

Theorem 3.6

Let (X, τ) be an S-closed L-fuzzy topological space and (Y, τ´) be a fully stratified iso-S-closed L-fuzzy topological space such that X is product related toY. Then X × Y is L-fuzzy iso-S-closed.

Proof:Let (X, τ) be an S-closed L-fuzzy topological space and (Y, τ´) be a fully stratified iso-S-closed L-fuzzy topological space and consider the projection map $P_Y : X\times Y \rightarrow Y$.

Let β be a countably compact, closed L-fuzzy subset of X×Y. $P_Y(\beta)$ is countably compact and closed L-fuzzy subset as P_Y being L-fuzzy continuous. By L-fuzzy iso-S-closedness of (Y,τ´), $P_Y(\beta)$ is S-closed L-fuzzy subset of (Y, τ´). Thus by 3.5, $X\times P_Y(\beta)$ is L-fuzzy S-closed. So β is L-fuzzy countably compact, closed subset of $X\times P_Y(\beta) \leq X \times Y$,and is S-closed L-fuzzy subset of X × Y. Hence X×Y is L-fuzzy iso-S-closed.

Definition 3.7An L-fuzzy topological space (X, τ) is called hereditarily iso-S-closed if every sub space of it is iso-S-closed.

Theorem 3.8

Let (X, τ) be a fully stratified iso-S-closed L-fuzzy topological space and (Y, τ´) be a hereditarily iso-S-closed L-fuzzy topological space such that X is product related to Y. Then X ×Y is L-fuzzy iso-S-closed.

Proof:Let (X, τ) be a fully stratified iso-S-closed L-fuzzy topological space and (Y, τ´) be a hereditarily L-fuzzy iso-S-closed space . Let us consider the projection map$P_Y : X \times Y \rightarrow Y$.

Let β be a countably compact, closed L-fuzzy subset of X×Y. Then $P_Y(\beta)$ is countably compact L-fuzzy subset of (Y, τ´). Since (Y, τ´) is hereditarily iso-S-closed L-fuzzy topological space then $P_Y(\beta)$ is L-fuzzy S-closed. Thus from 3.6, $X \times P_Y(\beta)$ is L-fuzzy iso-S-closed. Since β is a countably compact and closed subset of $X \times P_Y(\beta) \leq X \times Y$, β is L-fuzzy S-closed subset of X ×Y. Hence X × Y is L-fuzzy iso-S-closed.

Definition 3.9Let (X, τ) be an L-fuzzy topological space and $\lambda \in L^X$. The L-fuzzy subset λ is said to be iso- S*-closed if and only if every closed countably compact subset of λ is S*-closed.

If λ is the whole space, then the L-fuzzy topological space (X, τ) is also iso- S*-closed.

Theorem 3.10

If an L- fuzzy topological space (X, τ) is the union of a countable collection of closed and iso- S*-closed L-fuzzy subsets, then (X, τ) is L-fuzzy iso- S*-closed.

Proof: Suppose $X = \vee\mu_i$, where each μ_i is closed and iso-S*-closed L-fuzzy subset of X and let β be a closed and countably compact L-fuzzy subset of X. Let $p \in pr(L)$ and let $\{\gamma_i\}_{i\in J}$ be a family of semi-open L-fuzzy sets with $(\vee_{i\in J}(\gamma_i))(x) \nleq p$ for all $x \in X$ such that $\beta(x) \geq p'$.

For each i, $\beta\wedge\mu_i$ is a closed, countably compact L-fuzzy subset of μ_i. So it is S*-closed L-fuzzy subset, since each μ_i is L-fuzzy iso- S*-closed. By S*-closedness of $\beta\wedge\mu_i$, there exist a finite subset F of J with $(\vee_{i\in F}(cl *\gamma_i))(x) \nleq p$ for all $x\in X$ such that $(\beta\wedge\mu_i)(x) \geq p'$ i.e. $\beta(x) \geq p'$. Hence β is S*-closed L-fuzzy subset, which implies that X is L-fuzzy iso- S*-closed.

Theorem 3.11

Let $f:(X, \tau) \to (Y, \tau')$ be an L-fuzzy perfect and semi-irresolute mapping with $f^{-1}(y)$ is finite for every $y \in Y$, from an iso- S*-closed L-fuzzy topological space (X, τ) onto an L-fuzzy topological space (Y, τ´).Then (Y, τ´) is L-fuzzy iso- S*-closed.

Proof:Let β be a closed and countably compact L-fuzzy subset of (Y, τ´). Since f is L-fuzzy perfect map, then $f^{-1}(\beta)$ is closed and countably compact L-fuzzy subset of (X, τ). By L-fuzzy iso-S*-closednessof (X, τ), $f^{-1}(\beta)$ is L-fuzzy S*-closed. Since f is onto L-fuzzy semi-irresolute mapping with $f^{-1}(y)$ is finite for every $y \in Y$, then $f f^{-1}(\beta) = \beta$ is S*-closed [by 2.13] L-fuzzy subset in (Y, τ´). Hence (Y, τ´) is L-fuzzy iso- S*-closed.

Theorem 3.12

Let $f:(X, \tau) \to (Y, \tau')$ be an L-fuzzy perfect and irresolute mapping with $f^{-1}(y)$ is finite for every $y \in Y$, from an iso- S*-closed L-fuzzy topological space (X, τ) onto an L- fuzzy topological space (Y, τ´). Then (Y, τ´) is L-fuzzy iso- S*-closed.

Proof :Since every irresolute mapping is semi-irresolute[14] then the result is obvious from proposition [2.14].

Theorem 3.13

Let f :(X, τ) →(Y, τ´) be an L-fuzzy perfect and semi-irresolute mapping with f^{-1}(y) is finite for every y ∈Y, from an semi-iso-compact L-fuzzy topological space (X, τ) onto an L- fuzzy topological space (Y, τ´). Then (Y, τ´) is L-fuzzy iso-S∗-closed.

Proof:With the help of proposition 2.15, we can prove this theorem similarly as 3.11.

Theorem 3.14

If f: (X, τ) → (Y, τ´) be L-fuzzy perfect and semi weakly continuous mapping with f^{-1}(y) is finite for every y ∈Y, from an L-fuzzy semi-isocompact space (X, τ) onto an L- fuzzy topological space (Y, τ´), then (Y, τ´) is L-fuzzy iso- S∗-closed.

Proof:Let β be a closed and countably compact L-fuzzy subset of (Y, τ´). Since f is L-fuzzy perfect map, f^{-1}(β) is closed and countably compact L-fuzzy subset of (X, τ). By L-fuzzy semi- is compactness of (X, τ), f^{-1}(β) is L-fuzzy semi-compact. By 2.16, ff^{-1} (β) = β is S∗-closed L-fuzzy subset in (Y, τ´). Hence (Y, τ´) is L-fuzzy iso- S∗-closed.

Theorem 3.15

If (X, τ) and (Y, τ´) be two S*-closed L-fuzzy topological spaces such that X is product related to Y, then X × Y is L-fuzzy S∗-closed.

Proof:Let { $\lambda_i \times \beta_i$: i∈I}be an L-fuzzy cover of X × Y by semi-open L-fuzzy sets of X × Y, where λ_i's and β_i's are semi-open L-fuzzy sets in X and Y respectively. Then {λ_i: i∈I} and {β_i : i∈I}are L-fuzzy semi-open covers of X and Y respectively. As (X, τ) and (Y, τ´) are S*-closed L-fuzzy topological spaces then there exist finite subsets M and N of I such that, $(\vee_{i\in M}(cl{\ast}\lambda_i))$ (x) $\nleq$ p and $(\vee_{i\in N}(cl{\ast}\beta_i))$ (x) $\nleq$ pNow, {∨ cl∗ ($\lambda_i \times \beta_i$): i∈M ∨N} (x) = [∨{cl∗λ_i: i∈M ∨N}] (x) × [∨{cl∗β_i : i∈M ∨N}] (x)$\nleq$ p . Hence the proof.

Theorem 3.16

Let (X, τ) be an S∗-closed L-fuzzy topological space and (Y, τ´) be a fully stratified iso- S∗-closed L-fuzzy topological space such that X is product related toY. Then X × Y is L-fuzzy iso- S∗-closed.

Proof:Let (X, τ) be an S∗-closed L-fuzzy topological space and (Y, τ´) be a fully stratified iso- S∗-closed L-fuzzy topological space and consider the projection map P_Y : X×Y →Y.

Let β be a countably compact, closed L-fuzzy subset of X×Y. $P_Y(\beta)$ is countably compact and closed L-fuzzy subset as P_Y being L-fuzzy continuous. By L-fuzzy iso- S*-closedness of (Y, τ´), $P_Y(\beta)$ is S*-closed L-fuzzy subset of (Y, τ´). Thus by 3.15, $X\times P_Y(\beta)$ is L-fuzzy S*-closed. So β is L-fuzzy countably compact, closed subset of $X\times P_Y(\beta) \leq X \times Y$,and is S*-closed L-fuzzy subset of X × Y. Hence X×Y is L-fuzzy iso- S*-closed.

Definition 3.17An L-fuzzy topological space (X, τ) is called hereditarily iso- S*-closed if every sub space of it is iso- S*-closed.

Theorem 3.18

Let (X, τ) be a fully stratified iso- S*-closed L-fuzzy topological space and (Y, τ´) be a hereditarily iso- S*-closed L-fuzzy topological space such that X is product related to Y. Then X ×Y is L-fuzzy iso- S*-closed.

Proof:Let (X, τ) be a fully stratified iso- S*-closed L-fuzzy topological space and (Y, τ´) be a hereditarily L-fuzzy iso- S*-closed space . Let us consider the projection map$P_Y : X\times Y \to Y$.

Let β be a countably compact, closed L-fuzzy subset of X×Y. Then $P_Y(\beta)$ is countably compact L-fuzzy subset of (Y, τ´). Since (Y, τ´) is hereditarily iso- S*-closed L-fuzzy topological space then $P_Y(\beta)$ is L-fuzzy S*-closed. Thus from 3.16, $X \times P_Y(\beta)$ is L-fuzzy iso- S*-closed. Since β is a countably compact and closed subset of $X \times P_Y(\beta) \leq X \times Y$, β is L-fuzzy S*-closed subset of X ×Y. Hence X × Y is L-fuzzy iso- S*-closed.

Definition 3.19[12]An L-fuzzy topological space(X, τ) is said to be extremally disconnected if and only if $cl(\lambda) \in \tau$ for every $\lambda \in \tau$.

Kudri[14] established a relation among semi-compact space, s-closed and S*closedness in L-fuzzy topological spaces.

Proposition 3.20 [14]Semi-compactness ⇒S*-closedness⇒S-closedness.

Theorem 3.21

Let (X, τ) be an L-fuzzy topological space. Then the following relations hold.(X, τ) is Semi-iso-compact ⇒(X, τ) is iso-S*-closed⇒ (X, τ) is iso-S-closed.

Proof:The proof immediately follows from the proposition [3.20].

Theorem 3.22 [4]

Let (X, τ) be an extremally disconnected L-fuzzy topological space and $\lambda \in L^X$. Then the following are equivalent :

i) λ is almost compact [12].

ii) λ is nearly compact [14].

iii) λ is S-closed.

iv) λ is S∗-closed.

v) λ is SS-closed [4].

Theorem 3.23

Let (X, τ) be an extremally disconnected L-fuzzy topological space and $\lambda \in L^X$. Then the following are equivalent :

i) λ is weakly iso-compact [6].

ii) λ is nearly iso-compact [7].

iii) λ is iso-S-closed.

iv) λ is iso- S∗-closed.

v) λ is iso-SS-closed [9].

Proof:First of all, we show that (i) ⇒(ii). Suppose, (X, τ) is weakly iso-compact and extremally disconnected L-fuzzy topological space. Let λ be a regular closed, countably almost compact L-fuzzy subset of (X, τ). Since countably almost compact extremally disconnected L-fuzzy topological space is countably nearly compact then λ is countably nearly compact. As (X, τ) is L-fuzzy weakly isocompact, λ is almost compact and hence nearly compact (almost compact extremally disconnected L-fuzzy topological space is nearly compact). Hence (X, τ) is nearly isocompactL-fuzzy topological space.

(ii) ⇒(iii), (iii) ⇒(iv), (iv) ⇒(v) and (v) ⇒(i) can be proved similarly.

Corollary 3.24[4]Let (X, τ) be an extremally disconnected L-fuzzy topological space. If $\lambda \in L^X$ is compact then λ is S-closed(S∗-closed).

Corollary 3.25 Let(X, τ) be an extremally disconnected L-fuzzy topological space. If $\lambda \in L^X$ is iso-compact then λ is iso-S-closed (iso-S∗-closed).

Proof: Let λ be a closed countably compact L-fuzzy subset in (X, τ). Since it is isocompact then λ is compact. From 3.24, an extremally disconnected compact space is S-closed (S∗-closed) and so λ is S-closed (S∗-closed) and consequently it is iso-S-closed (iso-S∗-closed).

4. CL- ISO-S-CLOSEDNESSANDCL- ISO-S∗-CLOSEDNESSIN L-FUZZYTOPOLOGICAL SPACES

M.Sakai[16] introduced and studied CL-isocompactness(spaces in which closure of each countably compact subspace is compact) in classical topology. In this section considering the S-closedness and S∗-closednessin L-fuzzy topological spaces, a generalized stronger form of iso-S-closedness and iso-S∗-closednessare introduced and these new class of L-fuzzy topological spaces are called L-fuzzy CL-iso-S-closed and CL-isoS∗closedspaces. Some properties of these spaces are studied here.

Definition 4.1 Let (X, τ) be an L-fuzzy topological space. The L-fuzzy set $\lambda \in L^X$ is said to be L-fuzzy CL-iso-S-closed if the closure of each L-fuzzy countably compact subspace of λ is L-fuzzy S-closed. If λ is the whole space, then we say that the L-fuzzy topological space (X, τ) is L-fuzzy CL-iso-S-closed. Obviously every L-fuzzy CL-iso-S-closed spaces are L-fuzzy iso-S-closed.

Theorem 4.2

Let $f : (X, \tau) \to (Y, \tau')$ be an L-fuzzy perfect , almost continuous and almost open mapping with $f^{-1}(y)$ is finite for every y∈Y, from a CL-iso-S-closed space (X, τ) onto an L-fuzzy topological space (Y, τ´). Then (Y, τ´) is CL- iso-S-closed L-fuzzy topological space.

Proof: Letβ be an L-fuzzy countably compact subset of (Y, τ´). Since *f* is L-fuzzy perfect, $f^{-1}(\beta)$ is L-fuzzy countablycompact subset of (X, τ). As (X, τ) is L-fuzzy CL-iso-S-closed then cl ($f^{-1}(\beta)$) is L-fuzzy S-closed. Since *f* is L-fuzzy closed, continuous and onto then $f(\text{cl}(f^{-1}(\beta))) = \text{cl}(f(f^{-1}(\beta)))$ = cl (β), which implies cl (β) is L-fuzzy S-closed. Hence (Y, τ´) is L-fuzzy CL-iso-S-closed.

Theorem 4.3

Let (X, τ) be a fully stratified L-fuzzy iso-S-closed space and **(**Y, τ´) be an L-fuzzy CL- iso-S-closed space such that X is product related to Y. Then X × Y is L-fuzzy CL- iso-S-closed.

Proof: Let (X, τ) be a fully stratified iso-S-closed L-fuzzy topological space and (Y, τ´) be an L-fuzzy CL- iso-S-closed space. Let $P_X: X \times Y \to X$ and $P_Y: X \times Y \to Y$ be the projection maps.

Let β be an L-fuzzy countably compact subset of X × Y. Then $P_Y(\beta)$ is L-fuzzy countably compact. By L-fuzzy CL- iso-S-closedness of (Y, τ´), cl ($P_Y(\beta)$) is L-fuzzy S-closed in (Y, τ´). But $P_X(\beta)$ is L-fuzzy countably compact subset of X. Since X is fully stratified, then by 2.19, P_X

is L-fuzzy perfect and so is L-fuzzy closed. So $P_X(\beta)$ is L-fuzzy S-closed in X. Thus cl (β) is contained in the L-fuzzy S-closed space$P_X(\beta)\times$ cl ($P_Y(\beta)$) (by 3.5). Hence cl(β) is L-fuzzy S-closed, which follows that X × Y is L-fuzzy CL-iso-S-closed.

Definition 4.4An L-fuzzy topological space (X, τ) is called hereditarily CL-iso-S-closed if every subspace of it is CL-iso-S-closed.

Theorem 4.5

If an L-fuzzy topological space (X, τ) is hereditarily CL-iso-S-closed then (X, τ) is hereditarily iso-S-closed.

Proof:Since every CL-iso-S-closed L-fuzzy topological space is L-fuzzy iso-S-closed, the result follows immediately.

Definition 4.6Let (X, τ) be an L-fuzzy topological space. The L-fuzzy set $\lambda\in L^X$ is said to be L-fuzzy CL-iso- S∗-closed if the closure of each L-fuzzy countably compact subspace of λ is L-fuzzy S∗-closed. If λ is the whole space, then we say that the L-fuzzy topological space (X, τ) is L-fuzzy CL-iso- S∗-closed.

Obviously every L-fuzzy CL-iso- S∗-closed spaces are L-fuzzy iso- S∗-closed.

Theorem 4.7

Let f : (X, τ) → (Y, τ´) be an L-fuzzy perfect and semi-irresolute mapping with f^{-1}(y) is finite for every y∈Y, from a CL-iso- S∗-closed space (X, τ) onto an L-fuzzy topological space (Y, τ´). Then (Y, τ´) is CL- iso- S∗-closed L-fuzzy topological space.

Proof:Letβ be an L-fuzzy countably compact subset of (Y, τ´). Since f is L-fuzzy perfect, f^{-1} (β) is L-fuzzy countablycompact subset of (X, τ). As (X, τ) is L-fuzzy CL-iso- S∗-closed then cl (f^{-1} (β)) is L-fuzzy S∗-closed. Since f is L-fuzzy closed, continuous and onto then$f$ (cl (f^{-1} (β))) = cl (f (f^{-1} (β))) = cl (β), which implies cl (β) is L-fuzzy S∗-closed. Hence (Y, τ´) is L-fuzzy CL-iso- S∗-closed.

Theorem 4.8

Let (X, τ) be a fully stratified L-fuzzy iso-S∗-closed space and (Y, τ´) be an L-fuzzy CL- iso-S∗-closed space such that X is product related to Y. Then X × Y is L-fuzzy CL- iso-S∗-closed.

Proof:Let (X, τ) be a fully stratified iso- S∗-closed L-fuzzy topological space and (Y, τ´) be an L-fuzzy CL- iso- S∗-closed space. Let P_X: X × Y → X and P_Y: X × Y →Y be the projection maps.

Let β be an L-fuzzy countably compact subset of X × Y. Then $P_Y(\beta)$ is L-fuzzy countably compact. By L-fuzzy CL- iso- S∗-closedness of (Y, τ´), cl ($P_Y(\beta)$) is L-fuzzy S∗-closed in (Y, τ´). But $P_X(\beta)$ is L-fuzzy countably compact subset of X. Since X is fully stratified, then by 2.19, P_X is L-fuzzy perfect and so is L-fuzzy closed. So $P_X(\beta)$ is L-fuzzy S∗-closed in X. Thus cl (β) is contained in the L-fuzzy S∗-closed space$P_X(\beta)\times$ cl ($P_Y(\beta)$) (by 3.15). Hence cl(β) is L-fuzzy S∗-closed, which follows that X × Y is L-fuzzy CL-iso- S∗-closed.

Definition 4.9An L-fuzzy topological space (X, τ) is called hereditarily CL-iso- S∗-closed if every subspace of it is CL-iso- S∗-closed.

Theorem 4.10

If an L-fuzzy topological space (X, τ) is hereditarily CL-iso-S∗-closed then (X, τ) is hereditarily iso-S∗-closed.

Proof: Since every CL-iso- S∗-closed L-fuzzy topological space is L-fuzzy iso- S∗-closed, the result follows immediately.

CONCLUSION

There are**so** many compactness in literature. At present, many authors are working on various forms of compactness in fuzzy topology as well as L-fuzzy topological spaces. Our intention is to generalize these compactness in L-fuzzy topological spaces and to establish relations with other spaces. As a result we have defined concepts iso-S-closedness and iso- S∗-closedness and their stronger forms CL-iso-S-closedness and CL-iso- S∗-closedness and got some important relations with other spaces. We feel that with the help of these spaces some more new spaces will be developed in future.

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