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t- Pebbling number of a class of diameter two Graphs

M S Anil Kumar

Department of Mathematics, Mahatma Gandhi College Thiruvananthapuram
E-mail: anilkumar250365@gmail.com

ABSTRACT

For a graph G, define the pebbling number $f(G,v)$ to be the smallest integer m such that, for any distribution of m pebbles to the vertices of G, one pebble can be moved to a specified target vertex v and . $f(G) = \max_{v \in V(G)} f(G,v)$ The t- pebbling number $f_t(G)$ to be smallest integer in such that, for any distribution of m such that, for any distribution of m pebbles to the vertices of G, t pebbles can be moved to any specified target vertex. Pebbling number of large classes of graphs such as trees diameter two graphs, class zero graph etc. are calculated and is in Hulbert[2] . t-Pebbling number of graphs is another attraction in this area. The t-pebbling number of different kinds of graph and bounce two certain classes of graphs were discovered and is also seen in Hulbert[2]. This paper contains the generalized t-pebbling number of a class of diameter two graphs.

KEYWORDS: Pebbling, t-pebbling, diameter two graphs, pebbling number AMS Subject Classification 05C99

***Corresponding author**

M S Anil Kumar

Department of Mathematics,

Mahatma Gandhi College Thiruvananthapuram

E-mail: anilkumar250365@gmail.com

INTRODUCTION

The concept of pebbling in graphs[2] arose from an attempt by Lagarias and Saks to give an alternative proof of a theorem of Lemke and Kleitman. It is know that for any set N = {n_1, n_2,…..n_q} of q natural numbers, there is a nonempty index set I C {1, 2, ….,q} such that

$$q / \sum_{i\in I} ni \quad \text{and} \quad \sum_{i\in I} \gcd(q,ni) \leq q$$

However the proof was very complicated. It was the intension of Lagarias and Sakes to introduce Graph pebbling as a more intuitive vehicle for providing the theorem.

The purpose was accomplished by FRL Chung[1] in1989 when she published the first paper in graph pebbling.

2. Graph Pebbling

Suppose D is a distribution in which P pebbles are distributed on to the vertices of a graph G. Let $p(v)$ denotes the number of pebbles on the vertex v. A pebbling step consists of removing two pebbles from one vertex and then placing one pebble on to an adjacent vertex. We say a pebble can be moved to r, the 'target vertex', if we can apply pebbling steps repeatedly so that in the resulting distribution, r has a pebble on it.

Definition: Pebbling number[1]. For a graph G, we define the pebbling number f(G,v) to be the smallest integer m such that, for any distribution of m pebbles to the vertices of G, one pebble can be moved to a specified target vertex = v Again,

$$f(G) = \underset{v\in V(G)}{Max} f(G,\upsilon)$$

Definition[1] t-pebbling number: For a graph G, let f_t (G,v) denote the smallest integer m such that, if m pebbles are assigned to the vertices of G, then t pebbles can be moved to V. Again

$$f_t(G) = \underset{v\in V(G)}{Max} f_t(G,v)$$

3. t – pebbling number of a class of diameter two graphs.

Pachter et al[3] showed that, for any diameter two graph G, f(G) = n or n+1, where n is the number of vertices in G.

In this paper, we completely characterize the t – pebbling number of a large class of diameter two graphs.

Lemma 1: Let u be any vertex of G of degree at least two. Then

$$ft(G, u) \geq 4$$

Equality holds of deg u = n – 1

Proof: Let $v \in N[u]$

If p(u) = 2t-1, p (v) = 0, p(w) =1, $\forall \omega\varepsilon \vee (G)-\{u,v\}$ then t pebbles can not be place on to v.

Therefore, f_t (G, V) = 2t + n – 2 .

Lemma 2: Let v be any vertex of G of degree less than n – 1. Then f_t (G, v) $\geq$4t

Proof:- Let $u \notin N[v]$

If p (v) = 4t -1, then t pebbles cannot be moved to u.

Hence f_t (G, v) $\geq$ 4t

Corollary 3: For any vertex v of degree less than n – 1, we have f_t (G, v) $\geq$ max (2t+n-2, 4t)

We next discuss a case when equality holds in the above inequality.

Theorem 4: Let u and v be non adjacent vertices of G such that deg u = deg v = n – 2. Then

f_t (G, v) = f_t (G, v) = max (4t, 2t + n – 2).

Proof: When t = 1, the result reads as f (G, v) = max (4, n).

If n = 3, then G $\cong$ P_3, the path of three vertices. Also u and v are end vertices of G. It is known that f (G, v) = 4.

If , $n\geq 4$, the result reads as f (G, v) = n. Let D be a distribution of n pebbles on the vertices of G. If p (v) > 0 or $p(\omega)\geq 2$ for any vertex $\omega \in V(G)-\{u,v\}$ the result is proved. So, we may assume P (v) = 0, P(ω)$\leq$, $\forall \omega \in V(G)-\{u,v\}$

This implies $p(v)\geq 2$

If $p(\omega)\geq 1$ for at least one $\omega \in V(G)-\{u,v\}$ v can be pebbled using the path $u\omega v$

Otherwise p (x) = 0, $\forall x \in V(G)-\{u,v\}$

This implies p (u) = n $\geq$ 4

Therefore v can be pebbled. Thus the result is verified for t = 1

Let V_1 = V- {u, v} = {u_1, u_2, ….., u_{n-2}}

Case (a): 2t $\leq$ n – 2

In this case, we have to prove that f_t (G, v) = 2t + (n-2). We prove this by induction on t. Assume that the result is true for 1, 2, …., t – 1. Le D be the distribution of s = 2t + n – 2) pebbles on G. If either $p(v)\geq 1$ or $p(x)\geq 2$ for any , $x \in V_1$, we can move on pebble to v at the cost of at most two pebbles By induction we can move (t – 1) more pebbles to v. Therefore we may assume $p(v)=0$, $p(x)\le 1\ \forall\, x\in V_1. Let\ K=/\{x/x\in V_1, p(x)=1\}/$ Therefore p (u) = s – k.

Case (1):

In this case $(s-k)=2t+(n-2)-k\ge 2t.$ We can move one pebble each to t vertices in V_1. Then we can move t – pebbles to v.

Case (2): K < t.

In this case if we can move 2t – k pebbles to V1, We are through.

We have $(s-k)\ge 4t-2k$

We can move the pebbles in such a way that each of the k – occupied vertices have an even number of pebbles, the total number being 2t. Thus, we can move t pebbles to v. Therefore the result is true by induction for all t. such that 2t$\le$ n – 2.

Case (b): 2t = (n – 2) +1.

In this case we prove ft (G, v) $\le$ 4t. Let D be a distribution of 4t pebbles on G. Here 2(t – 1) = (n – 2) – 1. By case (s),

F_{t-1}(G, v) =4t – 3.

Clearly, we may assume p(v) = 0. Consider the following cases:

(i) $p(v)\ge 1$ (ii) $p(u)\ge 2$ and $p(w)\ge 1$ for some $\omega\in V_1$ (iii) $p(\omega)\ge 2$ for some $\omega\in V_1$

In each of the above cases, we can move one pebble to v (this includes case (i) also) at the cost of at most three pebbles: That is, after a pebbling process, we will have one pebble at v and at least (4t – 3) pebbles on V(G) [Excluding the one pebble at V]. The by induction,

We can move (t – 1) pebbles to v, making t pebbles in all at v.

Thus we may assume p(v) =0, $p(w)\le 1\ \forall\, x\in V_1$

Then $p(u)\ge 4t-(n-2)\ge 2t+1$

If now $p(\omega)\ge 1$ for some $\omega\in V_1$ we have proved (case (iii) above). Thus we may assume

$\sum_{w\in V_1} P(w)=0$, implying p(u) = 4t enabling us to move t pebbles to v.

Case (c) 2t $\ge$ n.

Again, we prove by induction on t. Whatever be the distribution of 4t pebbles on V, we can move one pebble to v at the cost of at most 4 pebbles. The remaining number of pebbles will be 4t – 4, with 2(2-1) $\ge$ n – 2. Therefore the result is true by induction.

Hence the proof of the theorem.

Corollary 5: f_t (G) = 4t if and only if either 2t$\ge$n – 2 and the minimum degree of all the vertices in G is n – 2 or G is complete and 2 t = n – 2.

Proof: Fist, suppose ft (G) = 4t. Lemma 1 implies $2t \geq n - 2$.

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