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Integral Solutions of The Octic Equation With Five Unknowns $(x-y)(x^3+y^3) = 12(w^2-p^2)T^6$

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ABSTRACT

The non-homogeneous octic equation with five unknowns represented by the Diophantine equation $(x-y)(x^3+y^3) = 12(w^2-p^2)T^6$is analyzed for its patterns of non-zero distinct integral solutions and seven different patterns of integral solutions are illustrated. A choice of interesting relations between the solutions and special numbers, namely, pyramidal numbers, pronic numbers, Stellaoctangular numbers, Gnomonic numbers, polygonal numbers, four dimensional figurate numbers are exhibited.

KEYWORDS: Octic non-homogeneous equation, Pyramidal numbers, Pronic numbers, Fourth, fifth and sixth dimensional figurate numbers.

Notations:

1. $t_{m,n}$- Polygonal number of rank n with size m
2. SO_n- Stella Octangula number of rank n
3. Pro_n- Pronic number of rank n.
4. G_n- Gnomonic number of rank n.
5. CP_n^m- Centered Pyramidal number of rank n with size m.
6. J_n- Jacobsthal number of rank n.
7. j_n- Jacobsthal-Lucas number of rank n.
8. ky_n- Kynea number of rank n
9. $FN_4^{\ 6}$- Four dimensional hexagonal figurate number of rank n.

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1. INTRODUCTION

The theory of diophantine equations offers a rich selection of absorbing problems. In particular, homogeneous and non-homogeneous equations of higher degree have aroused the interest of numerous mathematicians since archeological find [1-4]. In [5-12] heptic equation with three, four and five unknowns are analyzed. This communication analyses a non-homogeneous octic equation with five unknowns given by $(x-y)(x^3+y^3)=12(w^2-p^2)T^6$ for determining its infinitely many non-zero integer quintuples (x,y,w,p,T) satisfying the above equation are obtained. Various interesting properties among the values of x, y, p, w and T are offered.

2. METHOD OF ANALYSIS:

The non-homogeneous octic equation with five unknowns to be solved for its distinct non-zero integral solutions is

$$(x-y)(x^3+y^3)=12(w^2-p^2)T^6 \qquad \text{---} \qquad (1)$$

Introducing the linear transformations

$$\left.\begin{array}{ll} x=u+v, & w=uv+1 \\ y=u-v, & p=uv-1 \end{array}\right\} \qquad \text{---} \qquad (2)$$

in (1) leads to

$$u^2+3v^2=12T^6 \qquad \text{---} \qquad (3)$$

Different methods of obtaining the patterns of integers solutions to (1) are illustrated below.

Pattern1:

Imagine that $T=a^2+3b^2$ --- (4)

Mark 12 as

$$12=\frac{(3n+ni\sqrt{3})(3n-ni\sqrt{3})}{n^2} \qquad \text{---} \qquad (5)$$

using (4), (5) in (3) and applying the way of factorization, classify

$$u+i\sqrt{3}v=\left(3n+ni\sqrt{3}\right)(\alpha+i\sqrt{3}\beta) \qquad \text{---} \qquad (6)$$

Where $\alpha+i\sqrt{3}\beta=(a+i\sqrt{3}b)^6$ from which we have

$$\left.\begin{array}{l} \alpha=a^6-45a^4b^2+135a^2b^4-27b^6 \\ \beta=6a^5b-60a^3b^3+54ab^5 \end{array}\right\} \qquad \text{---} \qquad (7)$$

Equating the real and imaginary parts in (6), we have

$$\left.\begin{aligned} u &= 3\alpha - 3\beta \\ v &= \alpha + 3\beta \end{aligned}\right\} \qquad \text{---} \qquad (8)$$

Using (8) and (2) the values of x, y, w, p and T are given by

$$x = x(a,b) = 4\alpha$$

$$y = y(a,b) = 2\alpha - 6\beta$$

$$p = p(a,b) = 3\alpha^2 - 9\beta^2 + 6\alpha\beta - 1$$

$$w = w(a,b) = 3\alpha^2 - 9\beta^2 + 6\alpha\beta + 1$$

$$T = T(a,b) = a^2 + 3b^2$$

A few interesting properties observed are as follows.

1. $x(1,a) - 2y(1,a) - 1296\left({P_a}^5 * T_{4,a}\right) + 7776{FN_a}^4 + 45To_a + 1233Pro_a$

$$\equiv -270(mod\ 3285)$$

2. $y(a,1) + 2({CP_a}^6)^2 + 108{FN_a}^4 - 180T_{4,a} = 60$

3. $x(b,1) - 2(So_b * Star_b) - 576PT_b + 564\,OH_b + 276Pro_b - 195Gno_b = 195$

4. $T(2^b, 2^b)$is a square number.

Pattern 2:

Equ (3) can be written as

$$u^2 + 3v^2 = 12T^6 * 1 \qquad \text{---} \qquad (9)$$

Mark 1 as

$$1 = \frac{(n+ni\sqrt{3})(n-ni\sqrt{3})}{(2n)^2} \qquad \text{---} \qquad (10)$$

Substituting (4), (5) and (12) in (11) and employing the factorization method, define

$$u + i\sqrt{3}v = \frac{1}{2n^2}\left\{\left(3n + i\sqrt{3}n\right)\left(n + ni\sqrt{3}\right)\right\}\left(\alpha + i\sqrt{3}\beta\right) \qquad \text{---} \qquad (11)$$

Equating real and imaginary parts, we have

$$\left.\begin{aligned} u &= -6\beta \\ v &= 2\alpha \end{aligned}\right\} \qquad \text{---} \qquad (12)$$

Using (12) in (2) the values of x, y, w, p and T are given by

$x = x(a, b) = 2\alpha - 6\beta$

$y = y(a, b) = -2\alpha - 6\beta$

$w = w(a, b) = -12\alpha\beta + 1$

$p = p(a, b) = -12\alpha\beta - 1$

$T = T(a, b) = a^2 + 3b^2$

A few interesting properties observed are as follows.

1. $x(a, 1) - y(a, 1) - 4(Cub_a)^2 + 180BTq_a - 540T_{4,a} + 108 = 0$
2. $x(1, b) + w(1, b) + p(1, b) + 162(Nex_b * Gno_b) - 2430BTq_b - 3420{CP_b}^6 \equiv -160(mod\ 666)$
3. $y(a, 2) + 2(CP_b^6)^2 - 4320{FN_a}^4 + 3960T_{4,a} = 96$
4. $T(2^a, 2^{a-1}) - ky_a + j_{2a+1} - 9J_{2a-2} + 3 = 0$

Pattern 3:

Equ (3) can be written as

$$u^2 + 3v^2 = 4 * 3T^6 \quad --- \quad (13)$$

Mark 4 and 3 as

$$\left.\begin{aligned} 4 &= \frac{(n+ni\sqrt{3})(n-ni\sqrt{3})}{n^2} \\ 3 &= \frac{(3n+ni\sqrt{3})(3n-ni\sqrt{3})}{4n^2} \end{aligned}\right\} \quad --- \quad (14)$$

Following a similar method as in pattern 2 and the corresponding solutions of (1) are same as in pattern 2.

Pattern 4:

Equ (3) can be written as

$$u^2 + 3v^2 = 4 * 3T^6 * 1 \quad --- \quad (15)$$

Substituting (4), (10) and (14) in (15) and employing the method of factorization define,

$$u + i\sqrt{3}v = \frac{1}{4n^3}\{(n + ni\sqrt{3})(3n + ni\sqrt{3})(n + ni\sqrt{3})\}(\alpha + i\sqrt{3}\beta) \quad --- \quad (16)$$

Equating the real and imaginary parts, we have

$$\left.\begin{aligned} u &= -3\alpha - 3\beta \\ v &= \alpha - 3\beta \end{aligned}\right\} \qquad --- \qquad (17)$$

Using (17) in (2) the values of x, y, w, p and T are given by

$$x = x(a, b) = -2\alpha - 6\beta$$

$$y = y(a, b) = -4\alpha$$

$$w = w(a, b) = -3\alpha^2 + 6\alpha\beta + 9\beta^2 + 1$$

$$p = p(a, b) = -3\alpha^2 + 6\alpha\beta + 9\beta^2 - 1$$

$$T = T(a, b) = a^2 + 3b^2$$

A few interesting properties observed are as follows.

1. $2x(2, b) - y(2, b) + 648(ct_{2,b} * CP_b^3) - 7776FN_b^4 - 25056P_b^3 + 1404T_{18,b} - 9342Gno_b = 9342$

2. $x(a, 3) - So_a * To_a + 30(CP_a{}^6)^2 - 66(Ct_{2,a} * CP_a^3) - 17088PT_a + 26550TH_a - 16469T_{4,a} \equiv 144 \ (mod\ 4506)$

3. $T(2a^2 + 1, 2a^2) - 8(Pro_a * CS_a) - 12Ct_{2,a} + 10Gno_a + j_4 + 4 = 0$

4. $T(2^a, 2^{a-1}) - 9J_{2a-2} - j_{2a} - 2 = 0$

Pattern 5:

Let
$$\left.\begin{aligned} u &= 6U \\ v &= 2V \end{aligned}\right\} \qquad --- \qquad (18)$$

Using (18) in (3) we get,

$$3U^2 + V^2 = T^6 \qquad --- \qquad (19)$$

Using (4) and (19) and employing the method of factorization define,

$$V + i\sqrt{3}U = \alpha + i\sqrt{3}\beta \qquad --- \qquad (20)$$

Equating the real and imaginary parts in (20), we get

$$\left.\begin{aligned} V &= \alpha \\ U &= \beta \end{aligned}\right\} \qquad --- \qquad (21)$$

Substituting (21) in (18) and using (2), the non-zero distinct integral solutions (1) are

$$x = x(a, b) = 6\beta + 2\alpha$$

$$y = y(a, b) = 6\beta - 2\alpha$$

$$w = w(a, b) = 12\alpha\beta + 1$$

$$p = p(a, b) = 12\alpha\beta - 1$$

$$T = T(a, b) = a^2 + 3b^2$$

Pattern 6:

By means of (19) can be written as

$$3U^2 + V^2 = T^6 * 1 \qquad --- \qquad (22)$$

Using (4) and (10) and following the procedure as presented in pattern 2, the corresponding non-zero integral solutions to (1) are given by

$$x = x(a, b) = 4\alpha$$

$$y = y(a, b) = 2\alpha + 6\beta$$

$$w = w(a, b) = 3\alpha^2 - 6\alpha\beta - 9\beta^2 + 1$$

$$p = p(a, b) = 3\alpha^2 - 6\alpha\beta - 9\beta^2 - 1$$

$$T = T(a, b) = a^2 + 3b^2$$

3. CONCLUSION:

In this paper, we have made an effort to determine dissimilar patterns of non-zero distinct integer solutions to the non-homogeneous octic equation with five unknowns. As the octic equations are rich in variety, one may search for other forms of octic equation with variables greater than or equal to five and obtain their corresponding properties.

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