Edge Coloring In Intuitionistic Fuzzy Graph

R. Vinitha

Department of Mathematics
KG College of Arts and Science
Email: vinitha.r@kgcas.com

ABSTRACT

In this paper a new concept of coloring of intuitionistic fuzzy graphs has been introduced with illustrative examples.

KEYWORDS: Chromatic number, Edge coloring, Intuitionistic Fuzzy Graph (IFG), $(\alpha, \beta)$-cut.

*Corresponding author

R. Vinitha

Assistant Professor
Department of Mathematics
KG College of Arts and Science
Email: vinitha.r@kgcas.com
1. INTRODUCTION

The intuitionistic fuzzy sets are more practical and applicable in real life situations. Intuitionistic fuzzy set deal with incomplete information that is, degree of member function, non member function but not indeterminate and inconsistent information that exists definitely in many systems including belief system, decision support system etc.

2. PRELIMINARIES

2.1 DEFINITION

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ where $\forall u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \land \sigma(v)$.

2.2 DEFINITION

An Intuitionistic fuzzy set $A$ in a set $X$ is defined as an object of the form $A = \{X, \mu_A(X), \nu_A(X)\}$ where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ defined the degree of membership and degree of non membership of the element $x \in X$ respectively and for every $x \in X; 0 \leq \mu_A(X) + \nu_A(X) \leq 1$.

2.3 DEFINITION

Intuitionistic fuzzy graph is of the form $G = (V, F)$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_i : V \rightarrow [0,1]$ and $\nu_i : V \rightarrow [0,1]$ denote the degrees of membership and non membership of the element $v_i \in V$ respectively and $0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$ for every $v_i \in V (i = 1, 2, \ldots n)$

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0,1]$ and $\nu_2 : V \times V \rightarrow [0,1]$ are such that

\[
\mu_2(v_i, v_j) \leq \min[\mu_i(v_i), \mu_i(v_j)] \\
\nu_2(v_i, v_j) \leq \max[\nu_i(v_i), \nu_i(v_j)]
\]

and $0 \leq \mu_2(v_i, v_j) \lor \nu_2(v_i, v_j) \leq 1$ for each $(v_i, v_j) \in E$.

Here the triplet $(v_i, \mu_i, \nu_i)$ denote the vertex, the degree of membership and non membership of the vertex $v_i$. The triplet $(e_{ij}, \mu_2ij, \nu_2ij)$ denote the edge, the degree of membership and degree of non membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$. 
2.4 DEFINITION

Let A be intuitionistic fuzzy set of universe set X. Then \((\alpha, \beta)\)-cut of A is crisp set \((\alpha, \beta(A))\) of the Intuitionistic fuzzy set A is given by \((\alpha, \beta(A)) = \{x: x \in X \ni \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}\), where \((\alpha, \beta)\in [0,1]\) with \(\alpha + \beta \leq 1\).

3. COLORING OF INTUITIONISTIC FUZZY GRAPH

We consider an intuitionistic fuzzy graph with four vertices \(v_1, v_2, v_3, v_4\) and corresponding membership value 0.2,0.3,0.6,0.8 and corresponding non membership value 0.5,0.4,0.2,0.1. Graph consist of five edges \(e_1, e_2, e_3, e_4, e_5\) and corresponding membership value 0.2,0.2,0.6,0.3,0.2 and non membership value 0.5,0.5,0.1,0.4,0.5.

![Figure 1](image)

In this Intuitionistic fuzzy graph there are four \((\alpha, \beta)\)-cuts. They are (0.2,0.5), (0.6,0.1), (0.3,0.4).

For \((\alpha, \beta) = (0.2,0.5)\)

\[
\begin{pmatrix}
 v_1 & v_2 & v_3 & v_4 \\
 v_1 & 0.0 & 0.2 & 0.2 & 0.2 \\
 v_2 & 0.2 & 0.0 & 0.3 & 0.0 \\
 v_3 & 0.2 & 0.3 & 0.0 & 0.6 \\
 v_4 & 0.2 & 0.0 & 0.6 & 0.0 \\
\end{pmatrix}
\]

Matrix 1
The chromatic number of this graph is 3
For \((\alpha, \beta) = (0.6, 0.1)\)

\[
\begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.3 & 0.0 \\
0.0 & 0.3 & 0.0 & 0.6 \\
0.0 & 0.0 & 0.6 & 0.0 \\
\end{bmatrix}
\]

Matrix 4

\[
\begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.4 & 0.0 \\
0.0 & 0.4 & 0.0 & 0.1 \\
0.0 & 0.0 & 0.1 & 0.0 \\
\end{bmatrix}
\]
Matrix 5

\[
\begin{bmatrix}
 v_1 & v_2 & v_3 & v_4 \\
 v_1 & 0 & 0 & 0 \\
 v_2 & 0 & e_4 & 0 \\
 v_3 & 0 & e_4 & e_3 \\
 v_4 & 0 & 0 & e_3 \\
\end{bmatrix}
\]

Matrix 6

Matrix 7

\[
\begin{bmatrix}
 v_1 & v_2 & v_3 & v_4 \\
 v_1 & 0.0 & 0.0 & 0.0 & 0.0 \\
 v_2 & 0.0 & 0.0 & 0.3 & 0.0 \\
 v_3 & 0.0 & 0.3 & 0.0 & 0.0 \\
 v_4 & 0.0 & 0.0 & 0.0 & 0.0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
 v_1 & v_2 & v_3 & v_4 \\
 v_1 & 0.0 & 0.0 & 0.0 & 0.0 \\
 v_2 & 0.0 & 0.0 & 0.4 & 0.0 \\
 v_3 & 0.0 & 0.4 & 0.0 & 0.0 \\
 v_4 & 0.0 & 0.0 & 0.0 & 0.0 \\
\end{bmatrix}
\]

Figure 3

The chromatic number of this graph is 2.

For \((\alpha, \beta) = (0.3, 0.4)\)
Matrix 8

\[
E_3 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & e_4 & 0 \\
0 & e_4 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Matrix 9

\[
\begin{array}{c}
v_1(0.2,0.5) \\
\cdot \\
v_4(0.8,0.1) \\
\end{array}
\begin{array}{c}
v_2(0.3,0.4) \\
\cdot \\
v_3(0.6,0.2) \\
\end{array}
\begin{array}{c}
e_4(0.3,0.4) \\
\end{array}
\]

Figure 4

The chromatic number of this graph is 1.

For \((\alpha, \beta) = (1,1)\)

\[
\begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\end{bmatrix}
\]

Matrix 10

\[
\begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\end{bmatrix}
\]

Matrix 11
The chromatic number of this fuzzy graph is 0.

CONCLUSION

In this paper, coloring all the edges in Intuitionistic fuzzy graphs are introduced, edge chromatic number depends on \((\alpha, \beta)\)-cut value also analysed.

REFERENCE

1. Krassimir T. Atanassor, “Intuitionistic fuzzy sets, Fuzzy sets and systems”.
2. A.Prasanna, M.A.Rifayathali, S. Ismail Mohideen,” Strong Intuitionistic Fuzzy graph coloring”.
3. ArindamDey, DhrubajyothGhosh, Anita Pal,” Edge coloring of complement fuzzy graph”.
5. P.K.Sharma ,”\((\alpha, \beta)\)-cut of intuitionistic fuzzy groups”.
6. M.G.Karunambigai, R.Parvathi, R.Buvaneswari,”Arcs in Intuitionistic Fuzzy Graphs”.