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### **Six Dimensional Schwarzschild Brane Universe Sourced By Torsion**

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#### **ABSTRACT**

A local two form is revisited in a CFT on a  $D_5$ -brane world-volume to obtain an alternate space-time torsion curvature in a second order formalism. Interestingly, I explore a plausible torsion potential described by a two form, coupled to the one form, in a  $U(1)$  gauge theory on a  $D_5$ -brane. The non-linear  $U(1)$  gauge invariance in presence of a torsion is shown to define a geometric two form field strength. It is shown that the gauge dynamics in presence of torsion on a  $D_5$ -brane is approximated by the Dirac-Born-Infeld (DBI) action. We obtain some of the black hole geometries in presence of torsion and analyse their characteristic properties.

**KEYWORDS:** String Theory, D-brane, Kalb-Ramond potential, Schwarzschild Black hole, pair creation

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## INTRODUCTION

Einstein's General Theory of Relativity (GTR) has been the best theory to describe gravity classically for more than hundred years. The biggest problem with this theory is that it can't be quantised. Quantum gravity is still a big challenge to theoretical physicists. Modifications in GTR are the demand of time and a lot of people have attempted in different approaches to solve the problem of quantisation of gravity, e.g. <sup>1-10</sup>.

Einstein gravity describes black holes. In string theory, black holes have been obtained in a low energy limit of string effective action <sup>12-15</sup>. Quantum effects to Einstein gravity can also be addressed non-perturbatively using *s*-duality in superstring theories (10 dimensional). The type *IIA*-superstring theory in a strong coupling limit is known to incorporate an extra spatial dimensionon  $S^1$  and has been identified with the non-perturbative *M*-theory in eleven dimensions. Generically *M*-theory has been identified with the stringy vacua in various dimensions <sup>16, 17</sup>. In a low energy limit *M*-theory is known to describe an eleven dimensional supergravity. However a complete non-perturbative formulation of *M*-theory is still unknown.

Let me start with Non-linear string sigma model(NLSSM) action which is given by [11]

$$S = -\frac{T}{2} \int_{\Sigma} d^2\sigma \left( \sqrt{-h} h^{ab} g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \varepsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu - \alpha' \sqrt{-h} R \phi(X) \right) - \int_{\partial\Sigma} d\tau A_\mu(X) \partial_\tau X^\mu \quad (1)$$

Where  $T$  is the string tension and is inversely proportional to the slope parameter ( $\alpha'$ ) of Regge trajectory,  $h$  is the determinant of world-sheet metric  $h_{ab}$ ,  $\sigma$  and  $\tau$  are world-sheet parameters. The low energy effective space-time action is obtained from  $\beta$ -function equation.

In the past, one form potential ( $A_\mu$ ) gauge theory has been developed and understood very well (Quantum Electrodynamics) but two form potential ( $B_{\mu\nu}$ ) has not been explored much. Therefore, I have selected two form potential to explore its effects in various dimensions. Its gauge field is also nonlinear which is appropriate for gravitational geometries. The work presented here is basically, a six dimensional extension of <sup>24</sup>

It is known that the gauge invariance of the open string demands (refer any book on superstring theory e.g. String theory Vol-I,II by Polchinski):

$$2\pi\alpha' \mathcal{F}_{mn} = \bar{\mathcal{F}}_{mn} = (B_{mn} + 2\pi\alpha' F_{mn}) \quad (2)$$

Equation of motion of open string NLSSM action requires a mixed boundary condition:

$$(g_{\mu\nu} \partial_\sigma X^\nu + \bar{\mathcal{F}}_{\mu\nu} \partial_\tau X^\nu)|_{\partial\Sigma} = 0 \quad (3)$$

Dirichlet boundary condition requires a hypersurface at the end of open string known as *D*-

brane.  $D_p$ -brane dynamics is approximated by Dirac-Born-Infeld(DBI) action:

$$S_D = -T_D \int d^{p+1}x \sqrt{-(g + B + \bar{F})} \quad (4)$$

Open string parameter is given by<sup>11</sup>

$$G_{ij} = g_{ij} - (\bar{F}g^{-1}\bar{F})_{ij} \quad (5)$$

### **TORSION IN SUPERSTRING THEORY:**

In literature, authors have obtained generalized Riemann tensor build from the connection  $\Omega$  as given in<sup>12</sup>

$$R(\Omega)_{mnpq} = R(\omega)_{mnpq} + \nabla_p(\omega)T_{mnq} - \nabla_q(\omega)T_{mnp} + T^r_{mp}T_{rqn} - T^r_{mq}T_{rpn}, \quad (6)$$

Torsion  $T_{mnp}$  and scalar curvature  $R(\omega)$  are related as

$$R(\omega) = \frac{16}{3}\beta^2 H_{mnp}H^{mnp} = \frac{1}{3}T_{mnp}T^{mnp} \quad (7)$$

Where  $\omega$  is the spin connection and  $\beta = \frac{3\sqrt{2}}{8}e^{2\varphi}$ ,  $T_{mnp}$  is a totally antisymmetric tensor and  $H_{mnp}$  is the field-strength associated with  $B_{mn}$ .

### **PAIR CREATION**

The Schwinger mechanism<sup>19</sup> describes the pair production of electron and positron at a strong electromagnetic field vacuum. This mechanism is a non-perturbative phenomenon. Hawking used the similar idea to introduce black hole (Hawking) radiation which describes the pair creation at the black hole event horizon<sup>20</sup>. Bachas and Porrati applied the same idea for open strings pair creation<sup>22</sup>. Davis and Majumdar conjectured the cosmological creation of brane and anti-brane pair at the cosmological horizon. Interestingly, motivated authors have shown the emergent curvature on a pair of brane/anti-brane to be described by a geometric torsion in string-brane model<sup>23</sup>. The model is an attempt to explore the non-perturbative domain in quantum gravity underlying a string theoretic setup. Using the brane/anti-brane production tool by a two form has been performed to enhance the knowledge on M-theory<sup>24-26</sup>.

### **Geometric Torsion**

In effective curvature formalism, the totally antisymmetric tensor  $H_3$  is a connection in world volume theory. The covariant derivative is modified as given in [24]:

$$\mathcal{D}_i B_{\mu\nu} = \nabla_i B_{\mu\nu} + \frac{1}{2}(H_{\lambda\mu}{}^\rho B_{\rho\nu} - H_{\lambda\nu}{}^\rho B_{\rho\mu}) \quad (8)$$

Where  $\nabla$  is the GTR covariant derivative.

$H_3$  is modified to  $\mathcal{H}_3$  iteratively to incorporate all orders in Neveu-Schwarz (two forms)  $B_2$  as:

$$\mathcal{H}_{\mu\nu\lambda} = 3[\mu B_{\nu\lambda}] = H_{\mu\nu\lambda} + (H_{\mu\nu}{}^\alpha B_{\alpha\lambda} + \dots) + (H_{\mu\nu}{}^\beta B_{\beta\alpha} B^\alpha{}_\lambda + \dots). \quad (9)$$

### Effective torsion curvature

For a background open string metric and propagating geometric torsion,

$$[\mathcal{D}_\mu, \mathcal{D}_\nu]A_\lambda = \mathcal{K}_{\mu\nu\lambda}{}^\rho A_\rho, \quad (10)$$

Where

$$\mathcal{K}_{\mu\nu\lambda}{}^\rho \equiv \partial_\mu \mathcal{H}_{\nu\lambda}{}^\rho - \partial_\nu \mathcal{H}_{\mu\lambda}{}^\rho + \mathcal{H}_{\mu\lambda}{}^\sigma \mathcal{H}_{\nu\sigma}{}^\rho - \mathcal{H}_{\nu\lambda}{}^\sigma \mathcal{H}_{\mu\sigma}{}^\rho.$$

And

$$\mathcal{K}_{\mu\nu\lambda\rho} = -\mathcal{K}_{\nu\mu\lambda\rho} = -\mathcal{K}_{\mu\nu\rho\lambda}, \quad \mathcal{K}_{\mu\nu\lambda\rho} \neq \mathcal{K}_{\lambda\rho\nu\mu} \quad (11)$$

For a non-propagating torsion:  $\mathcal{K}_{\mu\nu\lambda\rho} \rightarrow R_{\mu\nu\lambda\rho}$ .

Other relevant curvature tensors are worked out to yield:

$$4\mathcal{K}_{\mu\nu} = -2\partial_\lambda \mathcal{H}^\lambda{}_{\mu\nu} + \mathcal{H}_{\mu\rho}{}^\lambda \mathcal{H}^\rho{}_{\lambda\nu}$$

And  $\mathcal{K} = -\frac{1}{4}\mathcal{H}_{\mu\nu\lambda}\mathcal{H}^{\mu\nu\lambda}$ . (12)

The 6-dimensional world-volume dynamics is described by <sup>16</sup>

$$S_{D_5} = \frac{1}{3C_2^2} \int d^6x \sqrt{G^{(NS)}} \mathcal{K}, \quad (13)$$

Where  $C_2$  signifies an appropriate coupling constant underlying a  $D_5$ -brane tension and

$$G_{\mu\nu}^{NS} = g_{\mu\nu} - B_{\mu\lambda}^{NS} g^{\lambda\rho} B_{\rho\nu}^{NS} \quad (14)$$

The trace of energy-momentum tensor becomes

$$\tilde{T} = \frac{(5-p)}{6} \mathcal{K} + \frac{(p+1)}{6} \tilde{\Lambda}, \quad (15)$$

With a gauge  $\tilde{\Lambda} = (3/\pi\alpha') + \mathcal{K}$ , the energy-momentum tensor becomes

$$(2\pi\alpha')\tilde{T}_{\mu\nu} = (\tilde{G}_{\mu\nu} - \frac{1}{4}\tilde{\mathcal{H}}_{\mu\lambda\rho}\mathcal{H}_\nu^{\lambda\rho}) \quad (16)$$

A geometric torsion in an effective metric may be redefined appropriately to imply that  $\tilde{T}_{\mu\nu}$  is indeed a precise source to the effective metric  $\tilde{G}_{\mu\nu}$ . An emergent metric sourced by a  $\tilde{T}_{\mu\nu}$  with on a  $D_5$ -brane in a type IIB superstring theory is given by

$$G_{\mu\nu}^{(NS)} = G_{\mu\nu} - \tilde{\mathcal{H}}_{\mu\lambda\rho}\mathcal{H}_\nu^{\lambda\rho} \quad (17)$$

### Two form ansatz

The gauge field ansatz satisfying the gauge field equation, may be given by

$$B_{\psi t}^{(NS)} = B_{\psi r}^{(NS)} = \frac{b}{\sqrt{(2\pi\alpha')}}$$

$$B_{\psi\phi} = \frac{P^3}{(2\pi\alpha')^{3/2}} (\sin^2\psi \cos\theta) \quad (18)$$

Where  $(b, P) > 0$  are constants. A nontrivial component of the field strength is worked out to

yield:

$$H_{\psi\theta\varphi} = \frac{P^3}{(2\pi\alpha')^2} (\sin^2\psi \sin\theta) \quad (19)$$

The parameter  $P$  incorporates a gauge theoretic torsion and ansatz satisfy the two form field equation:

$$\partial_\lambda H^{\lambda\mu\nu} + \frac{1}{2} (g^{\alpha\beta} \partial_\lambda g_{\alpha\beta}) H^{\lambda\mu\nu} = 0. \quad (20)$$

Spherically symmetric metric in six dimensions is given by

$$ds^2 = -dt^2 + (dr^2 + r^2 d\beta^2 + r_\beta^2 d\psi^2 + r_\beta^2 \sin^2\psi d\theta^2 + r_\beta^2 \sin^2\psi \sin^2\theta d\varphi^2) \quad (21)$$

Where  $r_\beta \equiv r \sin\beta$  and the angular coordinates are defined with  $(0 \leq \psi \leq \pi)$ ,  $(0 \leq \beta \leq \pi)$ ,  $(0 \leq \theta \leq \pi)$ ,

$(0 \leq \varphi < 2\pi)$ . They describe  $S^4$  symmetric vacuum configuration,

$$\begin{aligned} d\Omega_4^2 &= d\beta^2 + \sin^2\beta d\psi^2 + \sin^2\psi d\theta^2 + \sin^2\psi \sin^2\theta d\varphi^2, \\ &= d\beta^2 + \sin^2\beta d\Omega_3^2. \end{aligned} \quad (22)$$

For simplicity we identify some of the  $S^2$  symmetric line-elements within  $S^4$

$$\begin{aligned} d\Omega_\psi^2 &= d\psi^2 + \sin^2\psi d\theta^2, \\ d\Omega_\theta^2 &= d\theta^2 + \sin^2\theta d\varphi^2 \quad \text{and} \\ d\Omega_\beta^2 &= d\beta^2 + \sin^2\beta d\varphi^2. \end{aligned} \quad (23)$$

## SCHWARZSCHILD BRANE UNIVERSE

Now we work out the geometric torsion from the ansatzes (18) on a  $D_5$ -brane. Its non-trivial components are:

$$\begin{aligned} \mathcal{H}_{\theta\varphi}^\psi &= (2\pi\alpha')^{-1} \frac{P^3}{r_\beta^2} \sin^2\psi \sin\theta, \\ \mathcal{H}_{\theta\varphi}^t &= -\mathcal{H}_{\theta\varphi}^r = - (2\pi\alpha')^{-3/2} \frac{bP^3}{r_\beta^2} \sin^2\psi \sin\theta. \end{aligned} \quad (24)$$

From  $\hat{G}_{\mu\nu} = (g_{\mu\nu} - B_{\mu\lambda}^{(NS)} g^{\lambda\rho} B_{\rho\nu}^{(NS)})_{BG} \mp \frac{1}{2} \bar{\mathcal{H}}_{\mu\lambda\alpha} g^{\rho\alpha} g^{\lambda\sigma} \mathcal{H}_{\rho\nu\sigma}$  (25)

The emergent metric components are worked out (with  $2\pi\alpha' = 1$ ) to yield:

$$\begin{aligned} \hat{G}_{tt} &= - \left( 1 - \frac{b^2}{r_\beta^2} \pm \frac{b^2 P^6}{r_\beta^8} \right), \quad \hat{G}_{rr} = \left( 1 + \frac{b^2}{r_\beta^2} \mp \frac{b^2 P^6}{r_\beta^8} \right), \quad \hat{G}_{\psi\psi} = r^2, \\ \hat{G}_{\psi\psi} &= \left( 1 \mp \frac{P^6}{r_\beta^6} \right) r_\beta^2, \quad \hat{G}_{\theta\theta} = \hat{G}_{\psi\psi} \sin^2\psi, \quad \hat{G}_{\varphi\varphi} = \hat{G}_{\psi\psi} \sin^2\psi \sin^2\theta, \\ \hat{G}_{tr} &= \frac{b^2}{r_\beta^4} \hat{G}_{\psi\psi}, \quad \hat{G}_{t\psi} = \hat{G}_{r\psi} = \mp \frac{bP^6}{r_\beta^6} \end{aligned} \quad (26)$$

In a brane window, i.e.  $r_\beta > b$  and  $r_\beta > P$  with  $r_\beta^4 \gg b^4$  and  $r_\beta^{12} \gg P^{12}$ , the emergent patches may be approximated to define the brane universes:

$$ds^2 = -\left(1 - \frac{M}{r_\beta^2} \left[1 - \frac{P^6}{r_\beta^6}\right]\right) dt^2 + \left(1 - \frac{M}{r_\beta^2} \left[1 - \frac{P^6}{r_\beta^6}\right]\right)^{-1} dr^2 + r^2 d\beta^2 + \left(1 - \frac{P^6}{r_\beta^6}\right) r_\beta^2 d\Omega_3^2 + \frac{2M}{r_\beta^2} \left(1 - \frac{P^6}{r_\beta^6}\right) dt dr - \frac{2P^6\sqrt{M}}{r_\beta^6} (dt + dr) d\psi \quad (27)$$

And

$$ds^2 = -\left(1 - \frac{M}{r_\beta^2} \left[1 + \frac{P^6}{r_\beta^6}\right]\right) dt^2 + \left(1 - \frac{M}{r_\beta^2} \left[1 + \frac{P^6}{r_\beta^6}\right]\right)^{-1} dr^2 + r^2 d\beta^2 + \left(1 + \frac{P^6}{r_\beta^6}\right) r_\beta^2 d\Omega_3^2 + \frac{2M}{r_\beta^2} \left(1 + \frac{P^6}{r_\beta^6}\right) dt dr + \frac{2P^6\sqrt{M}}{r_\beta^6} (dt + dr) d\psi \quad (28)$$

Where  $M = b^2/(2\pi\alpha')^{3/2}$  may be identified with a mass parameter.

For  $M \neq 0$ , the geometries (27)-(28) under a limit  $r_\beta \rightarrow P$  modify to

$$ds_+^2 = -dt^2 + dr^2 + r^2 d\beta^2 - 2M (dt + dr) d\psi,$$

And

$$ds_-^2 = -\left(1 - \frac{2M}{r_\beta^2}\right) dt^2 + \left(1 - \frac{2M}{r_\beta^2}\right)^{-1} dr^2 + r^2 d\beta^2 + 2r_\beta^2 d\Omega_3^2 + \frac{4M}{r_\beta^2} dt dr + 2\sqrt{M} (dt + dr) d\psi. \quad (29)$$

The brane geometries in a regime  $(P - \delta P) < r < (P + \delta P)$  may be approximated

$$ds_+^2 = -dt^2 + dr^2 + r^2 d\beta^2,$$

And

$$ds_-^2 = -\left(1 - \frac{2M}{r_\beta^2}\right) dt^2 + \left(1 - \frac{2M}{r_\beta^2}\right)^{-1} dr^2 + r_\beta^2 d\Omega_4^2 + \frac{4M}{r_\beta^2} dt dr. \quad (30)$$

Interestingly the emergent black hole (27) in a specified brane regime is described by a  $(2 + I)$ -dimensional flat metric. The line-element may be identified with a  $D_2$ -brane within a  $D_5$ -brane in the formalism. The remaining brane universe, on a vacuum created  $(D\bar{D})_4$ -brane, may formally be identified with a  $(5+I)$ -dimensional Six dimensional Schwarzschild black hole

$$ds_-^2 = -\left(1 - \frac{2M}{r_\beta^2}\right) dt^2 + \left(1 - \frac{2M}{r_\beta^2}\right)^{-1} dr^2 + r_\beta^2 d\Omega_4^2. \quad (31)$$

For  $\beta = \pi/2$ , the brane universe is precisely described by a  $(4 + 1)$  - dimensional Schwarzschild black hole horizon at

$r \rightarrow r_s = 2M$  in Einstein vacuum. It may imply that a six dimensional brane universe may be viewed as an uplifted Schwarzschild vacuum in  $5D$  to a higher energy.

Generically the emergent geometries are created, at the horizon of a string black hole, on a vacuum pair of  $(D\bar{D})_4$ -brane by a two form quanta in a  $U(1)$  gauge theory.

$$ds^2 = -\left(1 - \frac{M}{r_\beta^2} + \frac{MP^6}{r_\beta^8}\right) dt^2 + \left(1 - \frac{M}{r_\beta^2} + \frac{MP^6}{r_\beta^8}\right)^{-1} dr^2 + r^2 d\beta^2 + \left(1 - \frac{P^6}{r_\beta^6}\right) r_\beta^2 d\Omega_3^2 -$$

$$ds^2 = - \left( 1 - \frac{M}{r_\beta^2} - \frac{MP^6}{r_\beta^8} \right) dt^2 + \left( 1 - \frac{M}{r_\beta^2} - \frac{MP^6}{r_\beta^8} \right)^{-1} dr^2 + r^2 d\beta^2 + \left( 1 + \frac{P^6}{r_\beta^6} \right) r_\beta^2 d\Omega_3^2 + \frac{2P^6\sqrt{M}}{r_\beta^6} dt d\psi \tag{32}$$

$$ds^2 = - \left( 1 - \frac{M}{r_\beta^2} - \frac{MP^6}{r_\beta^8} \right) dt^2 + \left( 1 - \frac{M}{r_\beta^2} - \frac{MP^6}{r_\beta^8} \right)^{-1} dr^2 + r^2 d\beta^2 + \left( 1 + \frac{P^6}{r_\beta^6} \right) r_\beta^2 d\Omega_3^2 + \frac{2P^6\sqrt{M}}{r_\beta^6} dt d\psi \tag{33}$$

Two distinct brane universes (32) and (33) in the quantum regime may be re-expressed using a renormalized mass or charge of a black hole

$$ds^2 = - \left( 1 - \frac{M_\mp}{r_\beta^2} \right) dt^2 + \left( 1 - \frac{M_\mp}{r_\beta^2} \right)^{-1} dr^2 + r^2 d\beta^2 + \frac{M_\mp}{M} r_\beta^2 d\Omega_3^2 - 2\sqrt{M} (M_\mp - M) dt d\psi \tag{34}$$

Where the masses  $M_\mp$  are computed at the event horizon ( $r_h = M_\mp \sin^{-1} \beta_0$ ) as

$$M_\mp = M \left( 1 \mp \frac{P^6}{r_\beta^6} \right) r_h = M \mp \frac{P^2}{M^2} \tag{35}$$

A decoupling of off diagonal patch simplifies the brane universes and the resultant geometries resemble to a Schwarzschild black hole in Einstein vacuum

$$ds^2 = - \left( 1 - \frac{M_\mp}{r_\beta^2} \right) dt^2 + \left( 1 - \frac{M_\mp}{r_\beta^2} \right)^{-1} dr^2 + r^2 d\Omega_4^2 \tag{36}$$

The torsion curvature might be seen to be created on a pair of  $(D\bar{D})_4$ -brane at the event horizon of the background black hole in Einstein vacuum by the quanta of a two form. The curvature scalar  $C_{\mu\nu\lambda\rho} C^{\mu\nu\lambda\rho}$  is independent of  $r$  which implies that (34) free from any curvature singularity. The Ricci scalar  $R$ :

$$R = \left( \frac{M}{r^4 \sin^2 \psi} + \frac{3M}{r^4} - \frac{35}{r^2} \right) \tag{37}$$

The near horizon correction may be:

$$ds^2 = - \left( 1 - \frac{M}{r_\beta^2} \right) dt^2 + \left( 1 - \frac{M}{r_\beta^2} \right)^{-1} dr^2 + r^2 d\Omega_4^2 \mp \frac{MP^2}{r_\beta^8} (dt^2 + dr^2 + \frac{r_\beta^4}{M} d\Omega_3^2) \tag{38}$$

And the Ricci scalar for the correction geometry in the regime is

$$R = - \frac{6}{r^2} \left( 6 - \frac{M}{r^2} \right) \tag{39}$$

In the near horizon limit  $r_\beta \rightarrow \sqrt{M}$ ,  $R$  simplifies to yield an AdS vacuum energy density  $\Lambda = -30/M$ . A  $D_5$ -brane is dual to a  $D$ -string in type IIB superstring theory. The black hole (with upper sign) is characterized by an inner horizon at  $r_- = (\sqrt{M}/\sin \beta) - \delta P$  and an event horizon at  $r_+ = (P/\sin \beta) + \delta P$ .

The angular velocities of the emergent black hole at the event horizon  $r_h$

$$\tilde{\Omega}_h^\psi = \frac{\tilde{G}_{t\psi}}{\tilde{G}_{\psi\psi}} \Big|_{r_h} = \mp \frac{1}{r_h^2 \sin^2 \beta} \left( \frac{P^6 \sqrt{M}}{(r_h^2 \sin^2 \beta)^3 + P^6 r_h^2 \sin^2 \beta \mp P^6} \right) \tag{40}$$

And

$$\tilde{\Omega}_h^\theta = \left. \frac{\tilde{G}_{t\theta}}{\tilde{G}_{\theta\theta}} \right|_{r_h} = \frac{r_h^2 \sin^2 \beta}{\sin \psi} \left( \frac{\tilde{P}^3 \sqrt{M}}{(r_h^2 \sin^2 \beta)^3 + P^6 r_h^2 \sin^2 \beta \mp P^6} \right). \quad (41)$$

### Cosmological constant

The curvature scalar sourced by a dynamical torsion in an effective curvature theory on a  $D_5$ -brane:

$$\mathcal{K} = - \frac{3P^6}{(4\pi\alpha')r_h^6}. \quad (42)$$

The effective curvature blows up in the limit  $r \rightarrow 0$ . But the curvature singularity is protected by the event horizon. The cosmological constant at  $r_h$  of the extremal black hole can be easily calculated to be

$$\frac{1}{3} \tilde{\Lambda}_{r_h} = \frac{1}{2\pi\alpha'} \left( 2 - \frac{P^6}{2b^6} \right). \quad (43)$$

An emergent  $AdS_2 \times S^4$  space-time on an effective  $D_5$ -brane enforces  $\tilde{\Lambda}_{r_h} < 0$ . The near horizon geometry of an emergent Schwarzschild black hole corresponds to an effective  $D_5$ -brane which is identified with asymptotic AdS.

## CONCLUSION

The work describes a geometric construction underlying a string-brane model to address an unresolved puzzle in quantum gravity the formalism was first developed in [26, 27]. In particular the model has been known to incorporate a non-perturbative quantum correction to the low energy string vacuum. Schwinger pair production mechanism was invoked to argue for a vacuum created gravitational pair of  $(4\bar{4})$ -brane at the cosmological horizon by a KR field quanta in the world-volume gauge theory on a  $D_5$ -brane. Interestingly quantum Schwarzschild like rotating brane geometries have been obtained. Also a brief analysis of their stability under an equipotential has been done showing that the quantum vacua reduce to the Schwarzschild black hole in a lowenergy limit.

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