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Equitable Power Domination Number of The Central Graph of Certain Graphs

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ABSTRACT:

For a graph $G(V,E)$ with vertex set V, a set $S \subseteq V$ is said to be a power dominating set (PDS), if every vertex $u \in V - S$ is observed by some vertices in S using the following rules: (i) if a vertex v in G is in PDS, then it dominates itself and all the adjacent vertices of v and (ii) if an observed vertex v in G has $k > 1$ adjacent vertices and if $k - 1$ of these vertices are already observed, then the remaining one non-observed vertex is also observed by v in G. A power dominating set $S \subseteq V$ of $G\ (V,\ E)$ is said to be an equitable power dominating set, if for every vertex $v \in V - S$ there exists an adjacent vertex $u \in S$ such that the difference between the degree of u and degree of v is less than or equal to 1, that is $|d(u) - d(v)| \leq 1$. The minimum cardinality of an equitable power dominating set of G is called the equitable power domination number of G and is denoted by $\gamma_{epd}(G)$. The central graph of a graph G, denoted by $C(G)$ is a graph obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G in $C(G)$. In this paper we investigate the equitable power domination number of the central graph of certain graphs.

KEYWORDS: Dominating set, Equitable power dominating set, Equitable dominating set, Equitable power domination number, Power dominating set, Power domination number, Central graph of a graph.

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INTRODUCTION

All graphs considered in this paper are simple, finite, connected, and undirected. The concept of domination in graphs was introduced by Hedetniemi and Laskar[1] in 1990, then the equitable domination and equitable domination number of certain graphs was studied by Swaminathan et al. In the year 1998, Haynes et al.[2,3] introduced the notion of power domination in graphs and power domination number of graphs. A dominating set[4] of a graph $G = (V, E)$ is a set S of vertices such that every vertex v in $V - S$ has at least one neighbor in S. The problem of finding a dominating set of minimum cardinality is an important problem that has been extensively studied. The minimum cardinality of a dominating set of G is called the domination number of G, denoted by $\gamma_d(G)$. A dominating set $S \subseteq V$ in $G(V, E)$ is said to be an equitable dominating set[5] if for every $v \in V - S$ there exists an adjacent vertex $u \in S$ such that the difference between the degree of u and degree of v is less than or equal to 1, that is $|d(u) - d(v)| \leq 1$. The minimum cardinality of an equitable dominating set of G is called the equitable domination number of G and is denoted by $\gamma_{ed}(G)$. Further, a set $S \subseteq V$ is said to be a power dominating set (PDS)[6] of G if every vertex $u \in V - S$ is observed by some vertices in S using the following rules:

(i) If a vertex v in G is in PDS, then it dominates itself and all the adjacent vertices of v.

(ii) If an observed vertex v in G has $k > 1$ adjacent vertices and if $k - 1$ of these vertices are already observed, then the remaining one non-observed vertex is also observed by v in G. The minimum cardinality of a power dominating set of G is called the power domination number of G, denoted by $\gamma_{pd}(G)$.

In 2017, Banu Priya et al.[7,8] introduced the notion of equitable power domination in graphs. A power dominating set $S \subseteq V$ in $G(V, E)$ is said to be an equitable power dominating set (EPD), if for every vertex $v \in V - S$ there exists an adjacent vertex $u \in S$ such that the difference between the degree of u and degree of v is less than or equal to 1, that is $|d(u) - d(v)| \leq 1$. The minimum cardinality of an equitable power dominating set of G is called the equitable power domination number of G and denoted by $\gamma_{epd}(G)$. One can note that an equitable power dominating set S of a graph G is not unique. Let G be a graph. The central graph $C(G)$ of G, is formed by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G in $C(G)$. In this paper we establish equitable power dominating set and equitable power domination number of the central graph of certain graphs.

Example 1:

Figure 1. **The equitable power domination number of a path P_4, $\gamma_{epd}(P_4) = 1$**

EQUITABLE POWER DOMINATION NUMBER OF THE CENTRAL GRAPH OF CYCLE, PATH, AND COMPLETE GRAPHS

Definition 1: [9]

A Path P_n is a graph whose vertices can be listed in the order $v_1, v_2, \dots, v_n$ such that the edges are $\{v_i, v_{i+1}\}$ where $i = 1, 2, \dots, n-1$.

Definition 2: [9]

A cycle is a path from a vertex back to itself (so the first and last vertices are not distinct).

Definition 3: [9]

A complete graph K_n is a graph in which any two distinct vertices are adjacent.

One can obtain the equitable power domination number of the central graph of a cycle C_n, for $n = 3, 4$ as it is shown in Figure 2. For $n \geq 5$ we have the following theorem.

Theorem 1:

Let $C(C_n)$ be the central graph of a cycle C_n.Then $\gamma_{epd}(C(C_n)) = n + 1$, for $n \geq 5$.

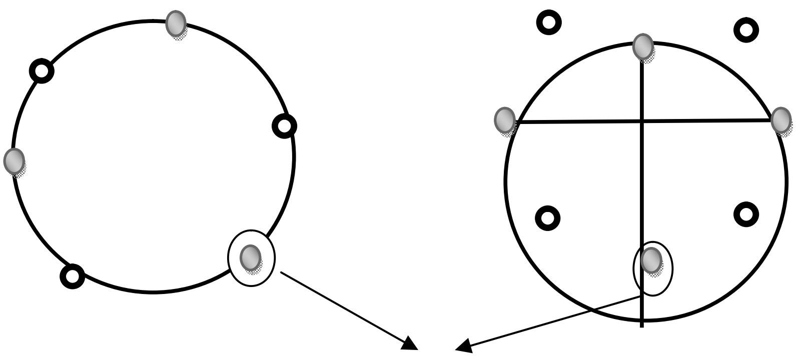


Figure 2. **(a) $\gamma_{epd}(C(C_3)) = 1$ and (b) $\gamma_{epd}(C(C_4)) = 1$**

Proof:

Let C_n, $n \geq 5$ be a cycle with $V(C_n) = \{u_1, u_2, \dots, u_n\}$ and with $E(C_n) = \{e_1, e_2, \dots, e_n\}$. Now obtain $C(C_n)$ as follows: $V(C(C_n)) = V_1 \cup V_2$, where $V_1 = V(C_n) = \{u_1, u_2, \dots, u_n\}$ and the set of newly added vertices $V_2 = v_i; 1 \leq i \leq n$. Note that the degree difference between u_i's and v_i's is greater than 1 for all i; $1 \leq i \leq n$. Hence the equitable power domination property does not hold good. Therefore one has to choose all the newly added vertices

$v_i; 1 \leq i \leq n$ to be in the equitable power dominating set S. Now one can observe that $d(u_i) = n - 1$ for all $1 \leq i \leq n$ in $C(C_n)$. So one can choose any one of these $u_i^{'}$s to be in S. Without loss of generality, let $u_1 \in S$. Now u_1 equitably power dominates all the vertices u_i; $3 \leq i \leq n - 1$. The remaining two non-observed vertices are u_2 and u_n. Moreover for the vertex u_3, the only non-observed vertex is u_n and it is u_2 for u_{n-1}. Interestingly, the vertex u_2 is observed by u_{n-1} and u_n is observed by u_3 by power domination property. Therefore $S = \{u_1\} \cup \{v_i ; 1 \leq i \leq n\}$ and $|S| = n + 1$.

Corollary 1:

Let $C(P_n)$ be the central graph of a path P_n. Then $\gamma_{epd}(C(P_n)) = n$ for $n \geq 5$.

Proof:

The proof is similar to Theorem 1.

Theorem 2: [7]

Let $K_n, n \geq 1$ be a complete graph on n vertices. Then $\gamma_{epd}(K_n) = 1$.

Theorem 3: [7]

Let G be a graph with n vertices. If $|d(u) - d(v)| \geq 2$ for any two vertices u, v of G that are adjacent, then $\gamma_{epd}(G) = n$.

Theorem 4:

Let $C(K_n)$ be the central graph of a complete graph K_n. Then $\gamma_{epd}(C(K_n)) = m + n$ for $n \geq 5$.

Proof:

Let K_n, $n \geq 5$ be a complete graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{e_1, e_2, \ldots, e_m\}$. Obtain the central graph of a complete graph $C(K_n)$ with vertex set $V(C(K_n)) = V_1 \cup V_2$, where $V_1 = V(K_n)$ and $V_2 = \{u_1, u_2, \ldots, u_m\}$. One can see that all the vertices in $C(K_n)$ are either of degree $n - 1$ or of degree 2 . So $|d(u_i) - d(v_i)| \geq 2$ for all i; $1 \leq i \leq n$. Therefore by Theorem 3, one has to choose all the vertices of $C(K_n)$ to be in the equitable power dominating set S. Thus $\gamma_{epd}(C(K_n)) = m + n$.

EQUITABLE POWER DOMINATION NUMBER OF THE CENTRAL GRAPH OF A COMPLETE BIPARTITE GRAPH

Definition 4: [9]

A complete bipartite graph, denoted $K_{m,n}$, is a simple bipartite graph with bipartition (X, Y) in which each vertex of X is joined to each vertex of Y.

Definition 5: [9]

A complete bipartite graph which is of the form $K_{1,n}$ is called a star.

One can see that the equitable power domination number of $C(K_{m,n})$, $\gamma_{epd}\left(C(K_{m,n})\right) = 1$ for $m + n < 5$. One such example is given in Figure 3. For $m + n \geq 5$, we have the following theorem.

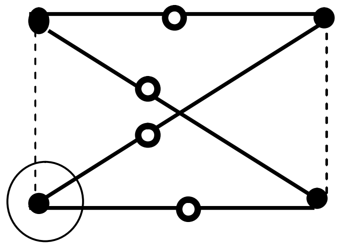


Figure 3. **The equitable power domination number of $C(K_{2,2}) = 1$**

Theorem 5:

Let $C(K_{m,n})$ be the central graph of a complete bipartite graph $K_{m,n}$. Then $\gamma_{epd}(C(K_{m,n})) = mn + 2$, for $m + n \geq 5$.

In order to prove Theorem 5, we first prove the following lemma.

Lemma 1:

Let $K_{m,n}$ be a complete bipartite graph. The central graph of a complete bipartite graph $C(K_{m,n})$ contains two complete graphs K_m and K_n as its subgraphs for all $m, n \geq 1$.

Proof:

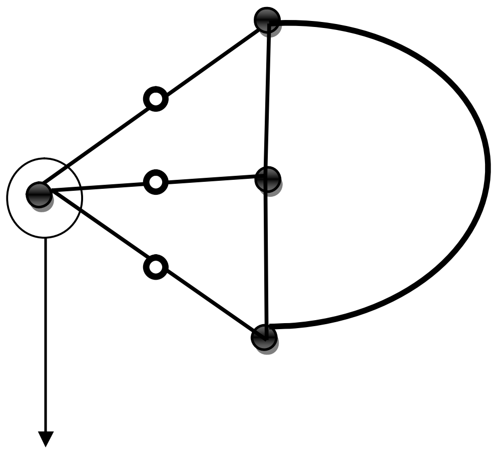
Let $K_{m,n}$ be a complete bipartite graph with two partite sets U_1 and U_2 where $U_1 = \{v_1, v_2, \dots, v_m\}$ and $U_2 = \{u_1, u_2, \dots, u_n\}$. The edge set $E(K_{m,n}) = \{e_1, e_2, \dots, e_{mn}\}$. The central graph of a complete bipartite graph is defined as follows: $V(C(K_{m,n})) = V_1 \cup V_2$, where $V_1 = V(K_{m,n})$ and $V_2 = \{w_1, w_2, \dots, w_{mn}\}$, the set of newly added vertices. The edge set $E\left(C(K_{m,n})\right) = E_1 \cup E_2 \cup E_3 \cup E_4$, where $E_1 = \{(v_i, v_j);\ 1 \leq i,j \leq m,\ i \neq j\}$, $E_2 = \{(u_i, u_j);\ 1 \leq i,j \leq n,\ i \neq j\}$, $E_3 = \{(v_i, w_j)\ ;\ 1 \leq i \leq m, 1 \leq j \leq mn\}$ and $E_4 = \{(w_i, u_j);\ 1 \leq i \leq mn, 1 \leq j \leq n\}$. It is clear that there are two partitions in a complete bipartite graph $K_{m,n}$ in which vertices in the same partition are not adjacent to each other. Therefore, by the definition of the central graph of a graph, there will be an edge between every pair of vertices in the partitions V_1 and V_2 of $K_{m,n}$ in $C(K_{m,n})$, which eventually forms the complete graphs of order m and n, respectively. Therefore $C(K_{m,n}) \supset K_m$ and $C(K_{m,n}) \supset K_n$. Hence the lemma.

Proof of Theorem 5:

Let $K_{m,n}$ be a complete bipartite graph with $m + n \geq 5$ and $C(K_{m,n})$ be the central graph of it. By Lemma 1, $C(K_{m,n})$ contains two complete graphs, say K_m and K_n. So by Theorem 2, it is enough to choose one vertex from each complete graphs, say $v_1 \in K_m$ and $u_1 \in K_n$, to construct an equitable power dominating set S. And also all w_i's, $1 \leq i \leq mn$, are of degree two in $C(K_{m,n})$ and no other adjacent vertices of w_i's satisfy the equitable property, therefore one has to include all the newly added vertices w_i's, $1 \leq i \leq mn$, in the equitable power dominating set S. Therefore $S = \{v_1\} \cup \{u_1\} \cup \{w_i \,;\, 1 \leq i \leq mn\}$ and $|S| = mn + 2$. Hence $\gamma_{epd}(C(K_{m,n})) = mn + 2$.

Corollary 2:

Let $C(K_{1,n})$ be the central graph of a star $K_{1,n}$. Then $\gamma_{epd}\big(C(K_{1,n})\big) = \begin{cases} 1, & \text{for } n \leq 3 \\ n+2, & \text{for } n \geq 4. \end{cases}$



vertex in the EPD Set S

Figure 4. The equitable power domination number of $C(K_{1,3}) = 1$

Definition 6: [10]

The Cartesian product of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ such that its vertex set is $V(G_1 \times G_2) = V(G_1) \times V(G_2) = \{(x, y) : x \in V(G_1),\ y \in V(G_2)\}$ and its edge set is defined as $E(G_1 \times G_2) = \{((x_1, x_2), (y_1, y_2)) : x_1 = y_1 \text{ and } (x_2, y_2) \in E(G_2)$ or $x_2 = y_2$ and $(x_1, y_1) \in E(G_1)\}$.

Defintion 7: [10]

The ladder graph L_n is defined by $L_n = P_n \times K_2$ where P_n is a path on n vertices and $\times$ denotes the Cartesian product and K_2 is a complete graph on two vertices.

One can easily obtain the equitable power domination number of the central graph of a ladder graph $P_{2,n}$, that is, $\gamma_{epd}\left(C(P_{2,n})\right) = 1$, for $n < 3$. One such example is given in Figure 5. For $n \geq 3$, we have the following theorem.

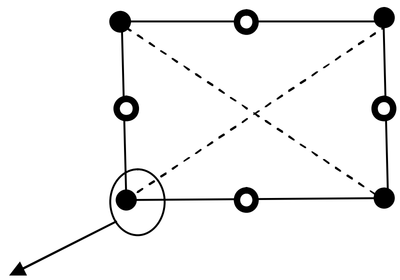


Figure 5. The equitable power domination number of $C(P_{2,n}) = 1$

Theorem 6:

Let $C(P_2 \times P_n)$ be the central graph of a ladder. Then $\gamma_{epd}(C(P_2 \times P_n)) = 3n - 1$, for $n \geq 3$.

Proof:

Let $P_2 \times P_n$ be a ladder with $V(P_2 \times P_n) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ and $E(P_2 \times P_n) = \{e_1, e_2, \dots, e_m\}$. Obtain the central graph of a ladder graph $P_2 \times P_n$ with vertex set $V(C(P_2 \times P_n)) = V_1 \cup V_2$, where $V_1 = V(P_2 \times P_n)$ and $V_2 = \{w_1, w_2, \dots, w_m\}$, the set of newly added vertices. One can note that the degree of all the newly added vertices in $C(P_2 \times P_n)$ is 2 and the degree of all vertices in V_1 is $2n - 1$. In obtaining the equitable power dominating set S, one has to choose all the newly added vertices to be in S since no vertices that are adjacent to the newly added vertices satisfy the required equitable property. Further, it is sufficient to choose any one of the remaining non-observed vertices, say u_1 to be in S which equitably power dominates all the vertices except its adjacent vertices namely v_1 and u_2 in $P_2 \times P_n$. But by the definition of power domination, v_3 dominates v_1 and u_3 dominates u_2. Thus $S = \{w_1, w_2, \dots, w_m\} \cup \{u_1\}$ and $|S| = 3n - 1$.

CONCLUSION

The equitable power domination number of the central graph of various classes of graphs have been determined. Establishing the same for other classes of graphs is an open problem and this is for future work. We also believe that the notion of equitable power domination in graphs may find applications in power networks.

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