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A Novel Shade to Obtain an Optimal Solution to a Fully Fuzzy Linear Programming Problem

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ABSTRACT

A different shade provided in this paper, is to find an optimal solution of a fully fuzzy linear programming problem. A different ranking method is provided for solving linear programming problem in a fuzzy uncertain environment. The proposed method is easy to understand and to apply for fully fuzzy linear programming in real life and flexible as compared to the existing one. Advantages of the proposed method over existing methods are discussed and to illustrate this method, a numerical example is solved and results are also discussed.

KEYWORDS: Fuzzy sets, Fuzzy number, Trapezoidal Fuzzy number, Fuzzy arithmetic, Fuzzy Ranking.

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1. INTRODUCTION

A typical mathematical program consists of a single objective function, representing either profits to be maximized or costs to be minimized and a set of constraints that circumscribe the decision variables. Decision making under fuzzy environment was first proposed by Bellman and Zadeh [1] The idea was adopted by several authors for solving fuzzy linear programming problems. Zimmermann[2] proposed an application of fuzzy optimization techniques to linear programming problem with single and multi-objective functions Rommel fanger et al[3]. Fang and Hu[4], Maleki et al[5], Maleki[6] and Khan et al[7] are studies where the objective functions, the decision variables, the technical coefficients and the constraints are fuzzy numbers respectively. All these studies considered fuzzy linear programming problems in which some of the parameters are crisps. Nasseri et al[8] and Amiri and Nasseri[9] are some examples utilizing ranking function method for solving linear programming problems without converting the problem to its crisp equivalent. The FLP problem is said to be a fully fuzzy linear programming (FFLP) problem if all parameters and variables are considered as fuzzy numbers. Recently, two methods have been introduced to solve the FFLP problems by Lotfi et al[10] and Kumar et al[11]. In the first method[10] the parameters of FFLP problem have been approximated to the nearest symmetric triangular fuzzy numbers. After that, a fuzzy optimal approximation solution has been achieved by solving a multi objective linear programming (MOLP) problem. As a resulting the optimal solution of FFLP is not exact. So, it is not reliable solution for decision maker. In the second method[11] an exact optimal solution is achieved using a linear ranking function. In this method, by the above ranking has been used to convert the fuzzy objective function to the crisp objective function. By this method, fuzziness of objective function has been neglected by the linear ranking function so by this method the originality of the problem will get changed . Now Our ultimate aim is to solve the given FFLPP without sacrificing the originality of the problem that means without converting them to a classical one. So in this paper we provide a new version of solving FFLPP. In general, most of the existing methods provide only crisp solutions for the fully fuzzy linear programming problem. In this paper we propose a simple method, for the solution of fully fuzzy linear programming problem without converting them in to classical fully fuzzy linear programming problem. The rest of this paper is organized as:In section 2, we recall the basic concepts and the results of trapezoidal fuzzy numbers and their arithmetic operations. In section 3, we define fully fuzzy linear programming problem and prove some of the important theorems, related results and also fuzzy version simplex algorithm is given. In section 4, numerical examples are provided to illustrate the methods proposed in this paper for the fully fuzzy linear programming problem without converting them to classical linear programming problem and the results obtained are discussed.

2 PRELIMINARIES

The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

2.1 Fuzzy number

A fuzzy set Ã defined on the set of feal numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics

(i) Ã is normal. It means that there exists an $x \in$ R such that $\mu_{\tilde{A}}(x) = 1$

(ii) $\tilde{A}$ is convex.

It means that for every $x_1, x_2 \in R$,

$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \lambda \in [0,1]$

(iii) $\mu_{\tilde{A}}$ is upper semi continous.

(iv)supp ($\tilde{A}$) is bounded in R.

2.2 Trapezoidal Fuzzy Number

A trapezoidal fuzzy number Ã is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$ and is defined by the membership

function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 0, & \text{otherwise} \end{cases}$$

We use *F*(**R**) to denote the set of all trapezoidal fuzzy numbers. Also if $m(\tilde{A})$ represents the mid point, $w(\tilde{A})$ represents the width, $\alpha = (a_2 - a_1)$ represents the left spread and $\beta = (a_4 - a_3)$ represents the right spread of the trapezoidal fuzzy number Ã, then the trapezoidal fuzzy number Ã can be represented by $\tilde{A} \approx (a_1, a_2, a_3, a_4) \approx (m(\tilde{A}), w(\tilde{A}), \alpha, \beta)$

2.3 Ranking of trapezoidal fuzzy numbers

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. That is, for every $\Re : F(R) \to R$ which maps each trapezoidal fuzzy number into a real number, where a natural order exists

For every trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, ranking function $\Re : F(R) \to R$ is defined by graded mean as

$$\Re\left(\tilde{A}\right) = \frac{(a_2 + a_3)}{2} + \frac{(\beta - \alpha)}{4}$$

For any two trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ we have the following comparison

(i) $\tilde{A} \succ \tilde{B}$ if and only if $\Re\left(\tilde{A}\right) \succ \Re\left(\tilde{B}\right)$

(ii) $\tilde{A} \prec \tilde{B}$ if and only if $\Re\left(\tilde{A}\right) \prec \Re\left(\tilde{B}\right)$

(iii) $\tilde{A} \approx \tilde{B}$ if and only if $\Re\left(\tilde{A}\right) \approx \Re\left(\tilde{B}\right)$

(iv) $\tilde{A} - \tilde{B} \approx \tilde{0}$ if and only if $\Re\left(\tilde{A}\right) - \Re\left(\tilde{B}\right) = 0$

A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4) \in F(R)$ is said to be positive if $\Re\left(\tilde{A}\right) \succ 0$

Also $\tilde{A} \approx \tilde{0}$ *if* $\Re\left(\tilde{A}\right) = 0$ and $\tilde{A} \prec \tilde{0}$ if $\Re\left(\tilde{A}\right) \prec 0$

If $\Re\left(\tilde{A}\right) = \Re\left(\tilde{B}\right)$ then the trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are said to be equivalent and it is denoted by $\tilde{A} \approx \tilde{B}$.

2.4 Arithmetic operations on trapezoidal fuzzy numbers

For any two arbitrary trapezoidal fuzzy numbers $\tilde{A} \approx (a_1, a_2, a_3, a_4) \approx (m(\tilde{A}), w(\tilde{A}), \alpha, \beta)$, the arithmetic operations are defined as follows:

(i). Addition: $\tilde{A} + \tilde{B} \approx (m(\tilde{A}) + m(\tilde{B}), \max\left(w(\tilde{A}), w(\tilde{B})\right), \max(\alpha_1, \alpha_2), \max(\beta_1, \beta_2))$

(ii). Subtraction: $\tilde{A} - \tilde{B} \approx (m(\tilde{A}) - m(\tilde{B}), \min\left(w(\tilde{A}), w(\tilde{B})\right), \min(\alpha_1, \alpha_2), \min(\beta_1, \beta_2)))$

(iii). Multiplication: $\tilde{A}\tilde{B} \approx (m(\tilde{A})\ m(\tilde{B}), \max\left(w(\tilde{A}), w(\tilde{B})\right), \max(\alpha_1, \alpha_2), \max(\beta_1, \beta_2))$

(iv). Division: $\tilde{A} / \tilde{B} \approx (m(\tilde{A}) / m(\tilde{B}), \min\left(w(\tilde{A}), w(\tilde{B})\right), \min(\alpha_1, \alpha_2), \min(\beta_1, \beta_2))$

3 MAIN RESULTS

3.1 General form of Fully Fuzzy Linear Programming

Optimize $(\max \text{ or } \min)\, \tilde{z} \approx (\tilde{c}_1 \tilde{x}_1 + \tilde{c}_2 \tilde{x}_2 + \tilde{c}_3 \tilde{x}_3 + \ldots + \tilde{c}_n \tilde{x}_n)$

subject to the following constraints,

$$
\begin{array}{l}
\tilde{a}_{11}\ \tilde{x}_1 + \tilde{a}_{12}\ \tilde{x}_2 + \tilde{a}_{13}\ x_3 + \ldots + \tilde{a}_{1n}\ \tilde{x}_n\ (\leq, \approx, \geq)\, \tilde{b}_1 \\
\tilde{a}_{21}\ \tilde{x}_1 + \tilde{a}_{22}\ \tilde{x}_2 + \tilde{a}_{23}\ \tilde{x}_3 + \ldots + \tilde{a}_{2n}\ \tilde{x}_n\ (\leq, \approx, \geq)\, \tilde{b}_2 \\
\tilde{a}_{31}\ \tilde{x}_1 + \tilde{a}_{32}\ \tilde{x}_2 + \tilde{a}_{33}\ \tilde{x}_3 + \ldots + \tilde{a}_{3n}\ \tilde{x}_n\ (\leq, \approx, \geq)\, \tilde{b}_3 \\
\ldots\ldots \\
\ldots\ldots \\
\ldots\ldots \\
\tilde{a}_{m1}\ \tilde{x}_1 + \tilde{a}_{m2}\ \tilde{x}_2 + \tilde{a}_{m3}\ \tilde{x}_3 + \ldots + \tilde{a}_{mn}\ \tilde{x}_n\ (\leq, \approx, \geq)\, \tilde{b}_m
\end{array}
\qquad \ldots\ldots(1)
$$

where

(i). $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ldots, \tilde{x}_n$ are fuzzy decision variables.

(ii). $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \ldots \tilde{c}_n$ are called fuzzy cost or fuzzy profit coefficients.

(iii). $\tilde{a}_{ij}$ $; i = 1, 2, 3 \ldots. m,\ j = 1, 2, 3 \ldots n$ are called structural fuzzy coefficients.

(iv). $\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \ldots\ldots, \tilde{b}_m$ represent fuzzy requirements or fuzzy availability of m constraints.

(v). The expression $(\leq, \approx, \geq)$ means that each constraint may take only one of the three possible forms.

(vi). The restrictions $\tilde{x}_j \geq 0$ simply implies that the $\tilde{x}_j$ must be non negative.

That is $(\max \text{ or } \min)\, \tilde{z} \approx \tilde{c}\,\tilde{x}$

Subject to $\tilde{A}\tilde{x} \approx \tilde{b}$, $\tilde{x} \geq \tilde{0}$, is a non negative fuzzy vector,

$$\text{where } \tilde{c} \approx \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \ldots \tilde{c}_n \in F(R)^{1Xn}$$
$$\tilde{x} \approx \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ldots, \tilde{x}_n \in F(R)^{nX1};$$
$$\tilde{b} \approx \tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \ldots\ldots, \tilde{b}_m \in F(R)^{mX1}$$

and

$$\tilde{A} \approx (\tilde{a}_{ij})_{m\times n} \approx \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \ldots. & \ldots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \ldots. & \ldots & \tilde{a}_{2n} \\ \ldots & \ldots & \ldots. & \ldots & \ldots \\ \ldots & \ldots & \ldots. & \ldots & \ldots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \ldots. & \ldots & \tilde{a}_{mn} \end{pmatrix} \in (F(R))^{m\times n}$$

Definition 3.1. The standard form of fully fuzzy linear programming problem is defined as $(\max \text{ or } \min)\, \tilde{z} \approx \tilde{c}\,\tilde{x}$ subject to $\tilde{A}\,\tilde{x} \approx \tilde{b}$ and $\tilde{x} \geq \tilde{0}$

Definition 3.2. Any $\tilde{x} \approx \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ldots, \tilde{x}_n \in F(R)^{nX1}$ where each $\tilde{x}_i \in F(R)$ which satisfies the constraints and non-negativity restrictions of (1) is said to be a *fuzzy feasible solution* to (1).

Definition 3.3. Let **S** be the set of all fuzzy feasible solutions of (1). A fuzzy feasible solution $\tilde{x}_0 \in S$ is said to be a *fuzzy optimum solution* to (1) if $\tilde{c}\,\tilde{x}_0 \geq \tilde{c}\,\tilde{x}$ for all $\tilde{x} \in S$ where $\tilde{c} \approx \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \ldots \tilde{c}_n$ and

$\tilde{c}\,\tilde{x} = \tilde{c}_1\,\tilde{x}_1 + \tilde{c}_2\,\tilde{x}_2 + \tilde{c}_3\,\tilde{x}_3 + \ldots + \tilde{c}_n\,\tilde{x}_n$.

Definition 3.4. Let $\tilde{x} \approx \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ldots, \tilde{x}_n$. Suppose $\tilde{x}$ solves $\tilde{A}\,\tilde{x} \approx \tilde{b}$. If all $\tilde{x}_j \approx (m(\tilde{x}_j), w(\tilde{x}_j), \alpha_j, \beta_j) \approx \tilde{0}$ for all j=1,2,...,n, then $\tilde{x}$ is said to be a *fuzzy basic solution*. If not, $\tilde{x}$ has some non-zero components, say $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ldots, \tilde{x}_k, 1 \leq k \leq n$. Then $\tilde{A}\,\tilde{x} \approx \tilde{b}$ can be written as:

$$\tilde{a}_1\,\tilde{x}_1 + \tilde{a}_2\,\tilde{x}_2 + \tilde{a}_3\,x_3 + \ldots + \tilde{a}_k\,\tilde{x}_k \approx \tilde{b}$$
$$\Rightarrow \tilde{a}_1\,\tilde{x}_1 + \tilde{a}_2\,\tilde{x}_2 + \tilde{a}_3\,x_3 + \ldots + \tilde{a}_k\,\tilde{x}_k + \tilde{a}_{k+1}\,\tilde{x}_{k+1} + \tilde{a}_{k+2}\,\tilde{x}_{k+2} + \ldots. + \tilde{a}_n\,\tilde{x}_n \approx \tilde{b}$$
$$\text{where } \tilde{x}_j \approx \tilde{0} \text{ for all } j = k+1, k+2, \ldots..n$$

If the columns $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \ldots.\tilde{a}_k$ corresponding to these non-zero components $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ldots, \tilde{x}_k$ are linearly independent, then $\tilde{x}$ is said to be a *fuzzy basic solution*.

Theorem 3.1. Let $\tilde{x}_{\tilde{B}} \approx \tilde{B}^{-1}\,\tilde{b}$ be a fuzzy basic feasible solution of (2). If for any column $\tilde{a}_j$ *in* $\tilde{A}$ *which is not in* $\tilde{B}$, the condition

$(\tilde{z}_j - \tilde{c}_j) \prec \tilde{0}$ *hold and* $\tilde{y}_{ij} \succ \tilde{0}$ *for some* $i, j \in \{1, 2, 3 \ldots..m\}$ then

it is possible to obtain a new fuzzy basic feasible solution by replacing one of the columns in $\tilde{B}$ by $\tilde{a}_j$

. ***Proof***

Suppose that $\tilde{x}_{\tilde{B}} \approx \left(\tilde{x}_{\tilde{B}_1}, \tilde{x}_{\tilde{B}_2}, \tilde{x}_{\tilde{B}_3} ....\tilde{x}_{\tilde{B}_m}\right)$ be a fuzzy basic feasible solution with k positive components such that $\tilde{B}\ \tilde{x}_{\tilde{B}} \approx \tilde{b}$ or $\tilde{x}_{\tilde{B}} \approx \tilde{B}^{-1}\ \tilde{b}$

$$where\ \tilde{x}_{\tilde{B}_i} \approx \left(m\left(\tilde{x}_{\tilde{B}_i}\right), w\left(\tilde{x}_{\tilde{B}_i}\right), \alpha_i, \beta_i\right) \succ \tilde{0}\ \ for\ i = 1, 2, ...k$$

$$and\quad \tilde{x}_{\tilde{B}_i} \approx \tilde{0}\ \ for\ \ i = k+1, k+2, ...m$$

Now equation $\tilde{B}\ \tilde{x}_{\tilde{B}} \approx \tilde{b}$ becomes

$$\sum_{i=1}^{k} \tilde{x}_{\tilde{B}_i} \tilde{b} + \sum_{i=k+1}^{m} \tilde{0}\,\tilde{b} \approx \tilde{b} ....(3)$$

Then for any column $\tilde{a}_j$ of $\tilde{A}$ which is not in $\tilde{\mathbf{B}}$, we write

$$\tilde{a}_j \approx \sum_{i=1}^{m} y_{ij}\, \tilde{b}_i \approx y_{1j}\, \tilde{b}_1 + y_{2j}\, \tilde{b}_2 + ... + y_{rj}\, \tilde{b}_r ... + y_{mj}\, \tilde{b}_m \approx y_j\, \tilde{B}$$

We know that if the basis vector $\tilde{b}_r$ for which $y_{rj} \neq 0$ is replaced by

$\tilde{b}_j$ of $\tilde{A}$ then the new set of vectors $\left(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, ..., \tilde{b}_{r-1}, \tilde{b}_j, \tilde{b}_{r+1}, ...\tilde{b}_m\right)$ still form a basis. Now for $\tilde{y}_{rj} \neq \tilde{0}$ and $r \le k$, we can write

$$\tilde{b} \approx \frac{\tilde{a}_j}{y_{rj}} - \sum_{i=1, i\neq r}^{m} \frac{y_{ij}}{y_{rj}} \tilde{b}$$

$$\approx \frac{\tilde{a}_j}{y_{rj}} - \sum_{i=1}^{k} \frac{y_{ij}}{y_{rj}} \tilde{b} - \sum_{i=k+1}^{m} \frac{y_{ij}}{y_{rj}} \tilde{b}$$

Equation (4) becomes

$$\sum_{i=1, i\neq r}^{k} \tilde{x}_{\tilde{B}_i} \tilde{b}_i + \tilde{x}_{\tilde{B}_r} \tilde{b}_r + \sum_{i=k+1}^{m} \tilde{0}\, \tilde{b} \approx \tilde{b}$$

$$\Rightarrow \sum_{i=1, i\neq r}^{k} \tilde{x}_{\tilde{B}_i} \tilde{b}_i + \tilde{x}_{\tilde{B}_r} \left(\frac{\tilde{a}_j}{y_{rj}} - \sum_{i=1}^{k} \frac{y_{ij}}{y_{rj}} \tilde{b}_i - \sum_{i=k+1}^{m} \frac{y_{ij}}{y_{rj}} \tilde{b}_i \right) + \sum_{i=k+1}^{m} \tilde{0}\, \tilde{b}_i \approx \tilde{b}$$

$$\Rightarrow \sum_{i=1, i\neq r}^{k} \tilde{x}_{\tilde{B}_i} \tilde{b}_i + \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \tilde{a}_j - \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \sum_{i=1, i\neq r}^{k} y_{ij} \tilde{b}_i - \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \sum_{i=k+1}^{m} y_{ij} \tilde{b}_i + \sum_{i=k+1}^{m} \tilde{0}\, \tilde{b}_i \approx \tilde{b}$$

$$\Rightarrow \sum_{i=1, i\neq r}^{k} \left(\tilde{x}_{\tilde{B}_i} - \frac{\tilde{x}_{\tilde{B}_r}}{\tilde{y}_{rj}} y_{ij} \right) \tilde{b}_i + \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \tilde{a}_j + \sum_{i=k+1}^{m} \left(\tilde{0} - \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \sum_{i=k+1}^{m} y_{ij} \right) \approx \tilde{b}$$

Since $\tilde{x}_{\tilde{B}_i} \approx \tilde{0}$; for i = k + 1; k + 2, ...m, we have

$$\sum_{i=1, i\neq r}^{k} \left(\tilde{x}_{\tilde{B}_i} - \frac{\tilde{x}_{\tilde{B}_r}}{\tilde{y}_{rj}} y_{ij} \right) \tilde{b}_i + \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \tilde{a}_j + \sum_{i=k+1}^{m} \left(\tilde{x}_{\tilde{B}_i} - \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \sum_{i=k+1}^{m} y_{ij} \right) \approx \tilde{b}$$

$$\sum_{i=1, i\neq r}^{m} \left(\tilde{x}_{\tilde{B}_i} - \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} y_{ij} \right) \tilde{b}_i + \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \tilde{a}_j \approx \tilde{b}$$

$$\sum_{i=1, i\neq r}^{m} \hat{\tilde{x}}_{\tilde{B}_i} \tilde{b}_i + \hat{\tilde{x}}_{\tilde{B}_r} \tilde{a}_j \approx \tilde{b} \quad \text{..............................(4)}$$

$$\text{where } \hat{\tilde{x}}_{\tilde{B}_i} \approx \tilde{x}_{\tilde{B}_i} - \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} y_{ij} \;; i \neq r$$

$$\hat{\tilde{x}}_{\tilde{B}_r} \approx \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}}$$

which gives a new fuzzy basic solution to $\tilde{A}\tilde{x} \approx \tilde{b}$. We shall show that this new fuzzy basic solution is also feasible. This requires that

$$\tilde{x}_{\tilde{B}_i} - \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} y_{ij} \geq \tilde{0} \,; i \neq r$$

$$\frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \geq \tilde{0}$$

$$\text{Select } y_{rj} \succ 0 \text{ such that } \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \approx \min \left(\frac{\tilde{x}_{\tilde{B}_i}}{y_{ij}} \,; y_{ij} \succ 0 \right)$$

$$\frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \leq \frac{\tilde{x}_{\tilde{B}_i}}{y_{ij}}$$

$$\left(m\left(\frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \right), w\left(\frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \right), \frac{\tilde{\alpha}_r}{y_{rj}}, \frac{\tilde{\beta}_r}{y_{rj}} \right) \leq \left(m\left(\frac{\tilde{x}_{\tilde{B}_i}}{y_{ij}} \right), w\left(\frac{\tilde{x}_{\tilde{B}_i}}{y_{ij}} \right), \frac{\tilde{\alpha}_i}{y_{ij}}, \frac{\tilde{\beta}_i}{y_{ij}} \right)$$

$$\left(m\left(\frac{\tilde{x}_{\tilde{B}_i}}{y_{ij}} \right), w\left(\frac{\tilde{x}_{\tilde{B}_i}}{y_{ij}} \right), \frac{\tilde{\alpha}_i}{y_{ij}}, \frac{\tilde{\beta}_i}{y_{ij}} \right) - \left(m\left(\frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \right), w\left(\frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \right), \frac{\tilde{\alpha}_r}{y_{rj}}, \frac{\tilde{\beta}_r}{y_{rj}} \right) \geq \tilde{0}$$

$$\left(m\left(\frac{\tilde{x}_{\tilde{B}_i}}{y_{ij}} \right) - m\left(\frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \right), \min\left(w\left(\frac{\tilde{x}_{\tilde{B}_i}}{y_{ij}} \right), w\left(\frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \right) \right), \min\left(\frac{\tilde{\alpha}_i}{y_{ij}}, \frac{\tilde{\alpha}_r}{y_{rj}} \right), \min\left(\frac{\tilde{\beta}_i}{y_{ij}}, \frac{\tilde{\beta}_r}{y_{rj}} \right) \right) \geq \tilde{0}$$

$$\Rightarrow \frac{\tilde{x}_{\tilde{B}_i}}{y_{ij}} - \frac{\tilde{x}_{\tilde{B}_r}}{y_{rj}} \geq \tilde{0}$$

Hence the new fuzzy basic solution is a fuzzy basic feasible solution.

Theorem 3.2. The set of optimal solution to the linear programming problem is convex.

Proof

Let $\tilde{s}_{F_0}$ denote the set of optimal solution

. If $\tilde{s}_{F_0}$ is empty or singleton , then it is convex.

Let $\tilde{s}_{F_0}$ contain more than one solution say $\tilde{x}_1, \tilde{x}_2 \in \tilde{S}_{F_0}$

Then $\tilde{c}\tilde{x}_1 \approx \tilde{c}\tilde{x}_2 \approx \max \tilde{z}$

Consider convex combination of $\tilde{x}_1, \tilde{x}_2$ as

$$\tilde{W}_0 \approx \lambda \tilde{x}_1 + (1-\lambda)\tilde{x}_2$$

$$\begin{aligned} \tilde{c}\tilde{W}_0 &\approx \tilde{c}\left(\lambda \tilde{x}_1 + (1-\lambda)\tilde{x}_2\right) \\ &\approx \lambda \tilde{c}\tilde{x}_1 + (1-\lambda)\tilde{c}\tilde{x}_2 \\ &\approx \lambda \max \tilde{z} + (1-\lambda) \max \tilde{z} \\ &\approx \max \tilde{z} \end{aligned}$$

$$\Rightarrow \tilde{W}_0 \in \tilde{S}_{F_0}$$

Hence $\tilde{s}_{F_0}$ is convex.

3.3 Fuzzy version of simplex algorithm

Step 1: Formulate the Fuzzy linear programming problem for the given data.

Step 2: Express the FFLP in the standard form by adding slack variables in the left hand side of less than or equal to constraint as well as to the objective function.

Optimize $(\max \text{ or } \min)\, \tilde{z} \approx (\tilde{c}_1\tilde{x}_1 + \tilde{c}_2\tilde{x}_2 + \tilde{c}_3\tilde{x}_3 + \ldots + \tilde{c}_n\tilde{x}_n) + \tilde{0}\tilde{s}_1 + \tilde{0}\tilde{s}_2 + \tilde{0}\tilde{s}_3 + \ldots + \tilde{0}\tilde{s}_m$

subject to the fuzzy linear constraints,

$$\begin{aligned} &\tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + \tilde{a}_{13}x_3 + \ldots + \tilde{a}_{1n}\tilde{x}_n + \tilde{s}_1 \approx \tilde{b}_1 \\ &\tilde{a}_{21}\tilde{x}_1 + \tilde{a}_{22}\tilde{x}_2 + \tilde{a}_{23}\tilde{x}_3 + \ldots + \tilde{a}_{2n}\tilde{x}_n + \tilde{s}_2 \approx \tilde{b}_2 \\ &\tilde{a}_{31}\tilde{x}_1 + \tilde{a}_{32}\tilde{x}_2 + \tilde{a}_{33}\tilde{x}_3 + \ldots + \tilde{a}_{3n}\tilde{x}_n + \tilde{s}_3 \approx \tilde{b}_3 \\ &\ldots\ldots \\ &\ldots\ldots \\ &\ldots\ldots \\ &\tilde{a}_{m1}\tilde{x}_1 + \tilde{a}_{m2}\tilde{x}_2 + \tilde{a}_{m3}\tilde{x}_3 + \ldots + \tilde{a}_{mn}\tilde{x}_n + \tilde{s}_m \approx \tilde{b}_m \\ &\text{and } \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ldots, \tilde{x}_n, \tilde{s}_1, \tilde{s}_2, \ldots\ldots \tilde{s}_m \geq \tilde{0} \end{aligned}$$

Where $\tilde{s}_1, \tilde{s}_2, \ldots\ldots \tilde{s}_m$ are slack variables which have been assigned zero fuzzy coefficient in the objective function.

Step 3: Set up tableau form of the fully fuzzy linear programming problem.

Before the initial simplex tableau can be set up obtain the initial

Feasible solution By setting $\tilde{x}_1 \approx \tilde{x}_2 \approx \tilde{x}_3 \approx \ldots \approx \tilde{x}_n \approx \tilde{0}$ such that in

the constraint set, we get

$\tilde{s}_1 \approx \tilde{b}_1, \tilde{s}_2 \approx \tilde{b}_2, \tilde{s}_3 \approx \tilde{b}_3 \ldots\ldots, \tilde{s}_m \approx \tilde{b}_m$

Step 4: Set up the initial tableau

Step 5: Test for optimality

If all the elements in the $\tilde{c}_j - \tilde{z}_j$ are negative or equal to zero then the current solution is optimal.If there exists some positive fuzzy number ,the current solution can be improved by removing one basic variable from the basis and replacing it by some non basic one.

Step 6: Revising simplex tableau At each iteration ,the simplex method moves from the current basic feasible solution to a better basic feasible solution.

(i)Determine the entering variable.

(ii) Determine the leaving variable.

(iii)Identify the pivotal number.

Step 7: construct a new simplex tableau.

(i) Compute new values for the pivotal row.

(ii) compute fresh values of the remaining rows as

$$\text{New row number} \approx \text{Old row number} - (\text{element in the pivot row} * \text{element in the pivot row}) \div \text{pivot number}$$

Step 8: Test the optimality

Step 9: If any of the numbers in $\tilde{c}_j - \tilde{z}_j$ row are positive, repeat the entire step 5 and 6 again.This process is repeated until an optimal solution has been obtained.

4 Numerical Examples

The following examples are taken from the paper solving fully fuzzy linear programming problem with inequality constraints by Amit Kumar, Jagdeep Kaur [3].

Example 4.1. Solve $\max \tilde{z} \approx (1,3,5,7)\tilde{x}_1 + (2,4,6,8)\tilde{x}_2$

Subject to the constraints,

$(0,2,4,5)\tilde{x}_1 + (1,2,3,4)\tilde{x}_2 \leq (1,10,27,48)$

$(2,4,5,6)\tilde{x}_1 + (2,4,6,8)\tilde{x}_2 \leq (4,20,45,80)$

Solution

First we convert the trapezoidal fuzzy number $\tilde{A} \approx (a_1, a_2, a_3, a_4) \approx (m(\tilde{A}), w(\tilde{A}), \alpha, \beta)$

$$\max \tilde{z} \approx (4,1,2,2)\tilde{x}_1 + (5,1,2,2)\tilde{x}_2$$

Subject to the constraints,

$$(3,1,2,1)\tilde{x}_1 + (2.5,.5,1,1)\tilde{x}_2 \leq (18.5,8.5,9,21)$$

$$(4.5,.5,2,1)\tilde{x}_1 + (5,1,2,2)\tilde{x}_2 \leq (32.5,12.5,16,35)$$

Standard form is

$$\max \tilde{z} \approx (4,1,2,2)\tilde{x}_1 + (5,1,2,2)\tilde{x}_2 + (0,0,0,0)\tilde{s}_1 + (0,0,0,0)\tilde{s}_2$$

Subject to the constraints,

$$(3,1,2,1)\tilde{x}_1 + (2.5,.5,1,1)\tilde{x}_2 + (1,0,0,0)\tilde{s}_1 \approx (18.5,8.5,9,21)$$

$$(4.5,.5,2,1)\tilde{x}_1 + (5,1,2,2)\tilde{x}_2 + (1,0,0,0)\tilde{s}_2 \approx (32.5,12.5,16,35)$$

The basic solution is

$$\tilde{s}_1 \approx (18.5,8.5,9,21)$$

$$\tilde{s}_2 \approx (32.5,12.5,16,35)$$

Table 1: Initial table

	$\tilde{c}_j$		$(4,1,2,2)$	$(5,1,2,2)$	$(0,0,0,0)$	$(0,0,0,0)$	
$\tilde{c}_{\tilde{B}}$	$\tilde{B}$	$\tilde{x}_{\tilde{B}}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	Row minimum
$(0,0,0,0)$	$\tilde{s}_1$	$(18.5,8.5,9,21)$	$(3,1,2,1)$	$(2.5,.5,1,1)$	$(1,0,0,0)$	$(0,0,0,0)$	$(7.4,.5,1,1)$
$(0,0,0,0)$	$\tilde{s}_2$	$(32.5,12.5,16,35)$	$(4.5,.5,2,1)$	$(5,1,2,2)$	$(0,0,0,0)$	$(1,0,0,0)$	$(6.5,1,2,2)$
	$\tilde{z}_j$	$(0,12.5,16,35)$	$(0,1,2,1)$	$(0,1,2,2)$	$(0,0,0,0)$	$(0,0,0,0)$	
		$\tilde{c}_j - \tilde{z}_j$	$(4,1,2,1)$	$(5,1,2,2)$	$(0,0,0,0)$	$(0,0,0,0)$	

$\tilde{x}_2$ is entering , $\tilde{s}_2$ is leaving

Table 2: First iteration table

	$\tilde{c}_j$		$(4,1,2,2)$	$(5,1,2,2)$	$(0,0,0,0)$	$(0,0,0,0)$
$\tilde{c}_{\tilde{B}}$	$\tilde{B}$	$\tilde{x}_{\tilde{B}}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$
$(0,0,0,0)$	$\tilde{s}_1$	$(2.25,.5,1,1)$	$(.75,.5,1,1)$	$(0,.5,1,1)$	$(1,0,0,0)$	$(0,0,0,0)$
$(5,1,2,2)$	$\tilde{x}_2$	$(6.5,1,2,2)$	$(.9,.5,2,1)$	$(1,1,2,2)$	$(0,0,0,0)$	$(.2,0,0,0)$
	$\tilde{z}_j$	$(32.5,1,2,2)$	$(4.5,1,2,2)$	$(5,1,2,2)$	$(0,1,2,2)$	$(1,1,2,2,)$
		$\tilde{c}_j - \tilde{z}_j$	$(-.5,.5,2,1)$	$(0,1,2,2)$	$(0,0,0,0)$	$(-1,0,0,0)$

All $\tilde{c}_j - \tilde{z}_j \leq \tilde{0}$

The current solution is optimal.

$\tilde{z} \approx (32.5,1,2,2)$

$\approx (29.5,31.5,33.5,35.5)$

$\tilde{x}_1 \approx (0,0,0,0)$

$\tilde{x}_2 \approx (6.5,1,2,2)$

$\approx (3.5,5.5,7.5,9.5)$

These results are sharper than the results obtained by the existing method.

Example 4.2

Solve $\max \tilde{z} \approx (0,1,2,3)\tilde{x}_1 + (2,3,4,5)\tilde{x}_2$

Subject to the constraints,

$(1,2,3,4)\tilde{x}_1 + (-3,2,5,10)\tilde{x}_2 \leq (-15,10,32,74)$

$(-2,3,5,6)\tilde{x}_1 + (4,5,7,8)\tilde{x}_2 \geq (-8,21,48,76)$

$(0,1,2,3)\tilde{x}_1 + (2,4,6,8)\tilde{x}_2 \approx (2,14,32,58)$

Solution

First we convert the trapezoidal fuzzy number $\tilde{A} \approx (a_1, a_2, a_3, a_4) \approx (m(\tilde{A}), w(\tilde{A}), \alpha, \beta)$

$\max \tilde{z} \approx (1.5,.5,1,1)\tilde{x}_1 + (3.5,.5,1,1)\tilde{x}_2$

Subject to the constraints,

$(2.5,.5,1,1)\tilde{x}_1 + (3.5,1.5,5,5)\tilde{x}_2 \leq (21,11,25,42)$

$(4,1,5,1)\tilde{x}_1 + (6,1,1,1)\tilde{x}_2 \geq (34.5,13.5,29,28)$

$(1.5,.5,1,1)\tilde{x}_1 + (5,1,2,2)\tilde{x}_2 \approx (23,9,12,26)$

Standard form is

$$\max \tilde{z} \approx (1.5,.5.1.1)\,\tilde{x}_1 + (3.5,.5,1,1)\,\tilde{x}_2 + (0,0,0,0)\,\tilde{s}_1 + (0,0,0,0)\,\tilde{s}_2$$
$$-(M,0,0,0)\,A_1 - (M,0,0,0)\,A_2$$

Subject to the constraints,

$$(2.5,.5,1,1)\,\tilde{x}_1 + (3.5,1.5,5,5)\,\tilde{x}_2 + (1,0,0,0)\,\tilde{s}_1 \approx (21,11,25,42)$$
$$(4,1,5,1)\,\tilde{x}_1 + (6,1,1,1)\,\tilde{x}_2 - (1,0,0,0)\,\tilde{s}_2 + (1,0,0,0)\,A_1 \approx (34.5,13.5,29,28)$$
$$(1.5,.5,1,1)\,\tilde{x}_1 + (5,1,2,2)\,\tilde{x}_2 + (1,0,0,0)\,A_2 \approx (23,9,12,26)$$

The basic solution is

$$\tilde{s}_1 \approx (21,11,25,42)$$
$$\tilde{A}_1 \approx (34.5,13.5,29,28)$$
$$\tilde{A}_2 \approx (23,9,12,26)$$

Table 3: Initial table

	$\tilde{c}_j$		$(1.5,.5,1,1)$	$(3.5,.5,1,1)$	$(0,0,0,$	$(0,0,0,0$	$(-M,0,0$	$(-M,0,0$
$\tilde{c}_{\tilde{B}}$	$\tilde{B}$	$\tilde{x}_{\tilde{B}}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{A}_1$	$\tilde{A}_2$
$(0,0,0,0$	$\tilde{s}_1$	$(21,11,25,42)$	$(2.5,.5,1,1)$	$(3.5,1.5,5,5$	$(1,0,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$
$(-M,0,0$	$\tilde{A}_1$	$(34.5,13.5,29,$	$(4,1,5,1)$	$(6,1,1,1)$	$(0,0,0,0)$	$(-1,0,0,0)$	$(1,0,0,0)$	$(0,0,0,0)$
$(-M,0,0$	$\tilde{A}_2$	$(23,9,12,26)$	$(1.5,.5,1,1)$	$(5,1,2,2)$	$(0,0,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$	$(1,0,0,0)$
	$\tilde{z}_j$	$(-57.5M,13.5,29,42$	$(5.5M,1,5,1)$	$(-11M,1.5,5,5)$	$(0,0,0,0)$	$(M,0,0,0)$	$(-M,0,0,0)$	$(-M,0,0,0)$
		$\tilde{c}_j - \tilde{z}_j$	$(1.5+5.5M,.5,1,1)$	$(3.5+11M,1,2,2)$	$(0,0,0,0)$	$(-M,0,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$

$\tilde{x}_2$ is entering

$\tilde{A}_2$ is leaving

Table 4: First iteration table

	$\tilde{c}_j$		$(1.5,.5,1,1)$	$(3.5,.5,1,1)$	$(0,0,0,$	$(0,0,0,0$	$(-M,0,0$	
$\tilde{c}_{\tilde{B}}$	$\tilde{B}$	$\tilde{x}_{\tilde{B}}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{A}_1$	Row minimum
$(0,0,0,0)$	$\tilde{s}_1$	$(4.9,1.5,5,5)$	$(1.45,.5,1,1)$	$(0,1.5,5,5)$	$(1,0,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$	$(3.38,.5,1,1)$
$(-M,0,0$	$\tilde{A}_1$	$(6.9,1,2,2)$	$(2.2,1,1,1)$	$(0,1,1,1)$	$(0,0,0,0)$	$(-1,0,0,0)$	$(1,0,0,0)$	$(3.14,1,1,1)$
$(3.5,.5,1,$	$\tilde{x}_2$	$(4.6,1,2,2)$	$(.3,.5,1,1)$	$(1,1,2,2)$	$(0,0,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$	$(15.3,.5,1,1)$
	$\tilde{z}_j$	$(-6.9M+16.1,1.5,5,5$	$(-2.2M+1.05,1,1,1$	$(3.5,1.5,5,5)$	$(0,0.5,1,1)$	$(M,0.5,1,1)$	$(-M,0.5,1,1)$	
		$\tilde{c}_j - \tilde{z}_j$	$(.45+2.2M,.5,1,1)$	$(0,0.5,1,1)$	$(0,0,0,0)$	$(-M,0,0,0)$	$(0,0,0,0)$	

$\tilde{x}_1$ is entering

$\tilde{A}_1$ is leaving

Table 5: Second iteration table

	$\tilde{c}_j$		$(1.5,.5,1,1)$	$(3.5,.5,1,1)$	$(0,0,0,$	$(0,0,0,0)$	
$\tilde{c}_{\tilde{B}}$	$\tilde{B}$	$\tilde{x}_{\tilde{B}}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	Row minimum
$(0,0,0,0)$	$\tilde{s}_1$	$(0.347,1,1,1)$	$(0,0.5,1,1)$	$(0,1,1,1)$	$(1,0,0,0)$	$(0,0,0,0)$	$(.5,1,1,1)$
$(-M,0,0$	$\tilde{x}_1$	$(3.14,1,1,1)$	$(1,1,1,1)$	$(0,1,1,1)$	$(0,0,0,0)$	$(-0.45,0,0,0$	-
$(3.5,.5,1,$	$\tilde{x}_2$	$(3.65,1,1,1)$	$(0,.5,1,1)$	$(1,1,1,1)$	$(0,0,0,0)$	$(0.135,0,0,0)$	$(27.5,1,1,1$
	$\tilde{z}_j$	$(17.52,1,1,1)$	$(1.5,1,1,1)$	$(3.5,1,1,1)$	$(0,0.5,1,1)$	$(-0.2,0.5,1,1)$	
		$\tilde{c}_j - \tilde{z}_j$	$(0,0.5,1,1)$	$(0,0,0,0)$	$(0.2,0,0,0$	$(-M,0,0,0)$	

$\tilde{s}_2$ is entering

$\tilde{s}_1$ is leaving

Table 6: Third iteration table

	$\tilde{c}_j$		$(1.5,.5,1,1)$	$(3.5,.5,1,1)$	$(0,0,0,0)$	$(0,0,0,0)$
$\tilde{c}_{\tilde{B}}$	$\tilde{B}$	$\tilde{x}_{\tilde{B}}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$
$(0,0,0,0)$	$\tilde{s}_2$	$(0.54,0,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$	$(1.54,0,0,0)$	$(1,0,0,0)$
$(1.5,0.5,$	$\tilde{x}_1$	$(3.38,1,1,1)$	$(1,0,0,0)$	$(0,0,0,0)$	$(0.693,0,0,0)$	$(0,0,0,0)$
$(3.5,.5,1,$	$\tilde{x}_2$	$(3.59,0,0,0)$	$(0,0,0,0)$	$(1,0,0,0)$	$(-0.21,0,0,0)$	$(0,0,0,0)$
	$\tilde{z}_j$	$(17.63,1,1,1)$	$(1.5,0.5,1,1)$	$(3.5,0.5,1,1)$	$(0.305,0.5,1,1)$	$(0,0.5,1,1)$
		$\tilde{c}_j - \tilde{z}_j$	$(0,0.5,1,1)$	$(0,0.5,1,1)$	$(-0.305,0,0,0$	$(0,0,0,0)$

All $\tilde{c}_j - \tilde{z}_j \leq \tilde{0}$

The current solution is optimal.

$\tilde{z} \approx (17.63,1,1,1)$

$\approx (15.63,16.63,18.63,19.63\ )$

$\tilde{x}_1 \approx (3.38,1,1,1)$

$\approx (1.38,2.38,4.38,5.38)$

$\tilde{x}_2 \approx (3.59,0,0,0)$

$\approx (3.59,3.59,3.59,3.59)$

These results are sharper than the results obtained by the existing method.

5 CONCLUSION

Based on the current study it can be concluded that it is better to use proposed version of solving Fully Fuzzy Linear Programming as it was compared to the existing one. In this paper, a new algorithm has been suggested to solve the FFLP problem. By simple examples and the obtained results of proposed algorithm (with Kumar's method) have been compared and shown the reliability and applicability of our algorithm

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