Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

On Some Generalisations of Fuzzy β-Ψ-Normal Spaces

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ABSTRACT

In this paper, the concept of fuzzy operation Ψ on a family of fuzzy β-open sets is introduced. The notions of fuzzy β-Ψ-almost β-normal and fuzzy β-Ψ-almost regular spaces are introduced and their properties are established. Also the concepts of fuzzy β-Ψ-nearly normal spaces and fuzzy β-Ψ-θ- normal spaces are introduced and some of their properties are discussed. Further, the notions of fuzzy β-Ψ-weak k-normal spaces are introduced and studied.

KEYWORDS: Fuzzy β-Ψ- almost β-normal spaces, fuzzy β-Ψ- almost regular spaces, fuzzy β-Ψ-nearly normal spaces and fuzzy β-Ψ-θ-normal spaces. 2010 AMS Subject Classification: 54A40, 03E72.

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INTRODUCTION

Csa´ sza´ r A´ [1] introduced the concept of an operation on the powerset P(X) of a topological space. Balasubramanian G[2] introduced the concept of fuzzy β-open sets. Amudhambigai B and Rowthri M[3] studied the notion of fuzzy operator ψ^* on a family of fuzzy α-open sets in a fuzzy topological systems in B-algebra. Normality plays an important role in general topology. Some generalised notions of normality such as almost normal[4], almost regular[5], k-normal[6], α-normal[7], β-normal[7], semi-normal[8], θ-normal[9] spaces were introduced and studied.

In this paper, the concept of fuzzy operation Ψ on a family of fuzzy β open sets is introduced. Further, the notions of fuzzy β-Ψ-almost β-normal spaces, fuzzy β-Ψ-almost regular spaces are introduced and their properties are established. Also the concepts of fuzzy β-Ψ-nearly normal spaces, fuzzy β-Ψ-θ-normal spaces and fuzzy β-Ψ-weak k-normal spaces are introduced and some of their properties are discussed.

PRELIMINARIES

Definition 2.1. Let (X, τ) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be a **fuzzy β-open set** if $\lambda \leq cl(int(cl(\lambda)))$. The complement of a fuzzy β open set is **fuzzy β-closed**.

Notation 2.1. Let (X, τ) be a fuzzy topological space. Then FβO(X) denotes the family of all fuzzy β-open sets of (X, τ) and FβC(X) denotes the family of all fuzzy β-closed sets of (X, τ).

Definition 2.2. Let (X, T) be a fuzzy topological space. Then X is called **fuzzy T_1 space** iff for each pair of distinct fuzzy points $x_\lambda$and y_α of X, there exist fuzzy open sets G, H containing x_λ and y_α respectively such that $y_\alpha \notin$ G and $x_\lambda \notin$ H.

Definition 2.3. A **fuzzy point** p in X is a fuzzy set with membership function $\mu_p(x) = y$, for $x = x_0$ *and* $\mu_p(x) = 0$, *otherwise* where $0 < y < 1$. p is said to have support x_0 and value y.

Definition2.4.[8] A fuzzy point x_λ is said to be **quasi-coincident** with a fuzzy set A, denoted by x_λqA, iff $\lambda > 1-A(x)$ or $\lambda + A(x) > 1$.

Definition2.5. A fuzzy set A is said to be quasi-coincident with another fuzzy set B, denoted by A q B, iff there exists $x \in$ X such that $A(x) > 1-B(x)$ or $A(x) + B(x) > 1$. If this is true, we also say that A and B are quasi-coincident (with each other) at x. It is clear that if A and B are quasi-coincident at x, both A(x) and B(x) are not zero andhence A and B intersect at x.

Definition 2.6. Let A and B be two fuzzy sets. Then A $\subset$B iff A and 1 − B are not quasi-coincident; denoted by A $\tilde{q}$ 1 –B, particularly, $x_\lambda \in$ Aiff x_λ is n o t quasi-coincident with 1 − A.

Definition 2.7. A topological space is said to be **almost normal** if for every pair of disjoint closed sets Aand Bone of which is regularly closed, there exist disjoint open sets U and V such that A⊆ U and B ⊆V.

Definition2.8. A topological space X is said to be **almost regular** if for every regularly closed A and a point $x \notin$ A, there exist disjoint open set U and Vsuch that A ⊆ U and $x \in$V.

Definition2.9. A topological space X is said to be α**-normal** if for any two disjoint closed subsets A and B of X there exist disjoint open subsets U and V of X such that A ∩ U is dense in A and B ∩ U is dense in B.

Definition2.10. A space X is β**-normal** if for any two disjoint closed subsets A and B of X there exist open subsets U and V such that A ∩ U is dense in A, B ∩ U is dense in B and $U \cap V = \emptyset$.

Definition2.11. A space X is said to be **semi-normal** if for every closed set A contained in an open set U, there exists a regularly open set V such that A ⊂ V ⊂ U.

Definition 2.12. A space is k**-normal** if for every pair of disjoint regularly closed sets E, F of X there exist disjoint open subsets U and V of X such that E ⊆ U and F⊆ V.

Definition2.13. A topological space X is said to be **nearly normal** if every pair of non empty disjoint sets one of which is δ-closed and the other is regularly closed are contained in disjoint open sets.

Definition 2.14. Atopological space X is said to be **weakly** θ**-normal** if every pair of disjoint θ-closed sets are contained in disjoint open sets.

FUZZY β-Ψ-ALMOST-β NORMAL SPACES AND FUZZY β-Ψ- ALMOST REGULAR SPACES

In this section, the concepts of fuzzy operation, fuzzy β-Ψ- normal spaces, fuzzy β- Ψ- almost β-normal spaces, fuzzy β-Ψ-almost regular spaces, fuzzy β-Ψ-weak θ-normal spaces, fuzzy β-Ψ-k-normal spaces and fuzzy β-Ψ-θ-normal spaces are introduced. Some interesting properties are discussed.

Definition 3.1. Let (X, τ) be a fuzzy topological space. A function $\Psi : F\beta O(X) \rightarrow I^X$ is called a **fuzzy operation** on FβO(X), if for each $0_X \neq \lambda \in F\beta O(X)$, $Fint(\lambda) \leq \Psi(\lambda)$ and $\Psi(0_X) = 0_X$.

Notation 3.1. Let (X, τ) be a fuzzy topological space. Then FO(X) denotes the set of all fuzzy operations on FβO(X).

Remark 3.1. It is facile to examine that some examples of fuzzy operations on FβO(X) with the familiar fuzzy operators like *Fint*, *Fint(Fcl)*, *Fint(Fcl*$_\delta$*)*, *Fcl(Fint)*, *Fint(Fcl(Fint))*, *Fcl*(*Fint*(*Fcl*)).

Example 3.1. Let X = {a, b} and τ = { 0_X, 1_X, λ_1, λ_2} where λ_i: X $\rightarrow$[0, 1] for i=1,2 is defined as follows λ_1(a)=0.4, λ_1(b)=0.6; λ_2(a)=0.8, λ_2(b)=0.7. Clearly, (X,τ) is a fuzzy topological space. Let Ψ=*Fcl(Fint)*. Consider the fuzzy β open set μ where $\mu(a)$ = 0.5, $\mu(b)$ = 0.6. Then *Fcl(Fint*(μ)) = 1_X. Also *Fint*(μ) = λ_1. Hence *Fint*(μ) <$\Psi(\mu)$ and $\Psi(0_X) = 0_X$. Similarly, *Fint*(μ_i) ≤ $\Psi(\mu_i)$, $i \in$ I and $\Psi(0_X)$ = 0_X for all $\mu_i \in$ FβO(X). Thus Ψ is a fuzzy operation on FβO(X).

Definition3.2. Let (X,τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). Let $\lambda \in I^X$ be any fuzzy β open set. Then λ is called a fuzzy β-**Ψ-open** if $\lambda \leq \Psi(\lambda)$. The complement of a fuzzy β-Ψ-open set is a fuzzy β-Ψ-closed set.

Notation 3.2. Let (X, τ) be a fuzzy topological space. The family of all fuzzy β-Ψ- open sets and fuzzy β-Ψ-closed sets are denoted by F$\beta\Psi$O(X) and F$\beta\Psi$C(X) respectively.

Example 3.2. Let X = {a, b}. As in Example 3.1 define the fuzzy topology τ = {0_X, 1_X, λ_1, λ_2}. Clearly (X, τ) is a fuzzy topological space. Let Ψ = *Fcl(Fint)*. Consider the fuzzy β open set μ where $\mu(a)$ = 0.6, $\mu(b)$ = 0.6. Then *Fcl(Fint*(μ)) = 1_X. Henceμ < $\Psi(\mu)$.This implies that μ is a fuzzy β-Ψ- open set. Also (1−μ)=γ where $\gamma(a)$= 0.4 and $\gamma(b)$=0.4 and so (1−μ) is a fuzzy β-Ψ-closed set.

Definition 3.3. Let (X, τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). Let λ be any fuzzy set. Then the **fuzzy** β**-Ψ-interior** of λ denoted by Fβ-Ψ-*int*(λ) is defined as Fβ-Ψ-*int*(λ) = ∨{μ; $\mu \in$ F$\beta\Psi$O(X) and $\lambda \geq \mu$}.

Definition 3.4. Let (X, τ)be a fuzzy topological space and $\Psi \in$ F*O*(X). Let λ be any fuzzy set. Then the **fuzzy** β**-Ψ-closure** of λ denoted by Fβ-Ψ-*cl*(λ) is definedas Fβ-Ψ-*cl*(λ) = ∧{μ; $\mu \in$ F$\beta\Psi$C(X) and $\lambda \leq \mu$}.

Definition 3.5. Let (X, τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). Let λ be any fuzzy set. Then λ is called a **fuzzy** β**-Ψ- regular open** (resp.**fuzzy** β**-Ψ- regular closed**) set of (X, τ) if λ = Fβ-Ψ-*int*(Fβ-Ψ-*cl*(λ))(resp.λ=Fβ-Ψ-*cl*(Fβ-Ψ-*int*(λ))).

Notation 3.3. Let (X, τ) be a fuzzy topological space. The family of all fuzzy β-Ψ- regular open sets and fuzzy β-Ψ-regular closed sets are denoted by F$\beta\Psi$RO(X) and F$\beta\Psi$RC(X) respectively.

Definition3.6. Let (X,τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). Then (X,τ) is said to be a fuzzy β**-Ψ- almost normal space** if for any two fuzzy β-Ψ- closed sets λ, μ one of which is fuzzy β-Ψ-regular closed such that $\lambda \tilde{q} \mu$, there exist fuzzyβ-Ψ-open sets δ, γ with $\delta \tilde{q} \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$.

Definition 3.7. Let (X, τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). Then (X,τ) is said to be a fuzzy β**-Ψ- almost** β**-normal space** if for any two fuzzy β-Ψ- closed sets λ, μ one of which is fuzzy

β-Ψ- regular closed such that $\lambda \tilde{q} \mu$, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \tilde{q} \gamma$ such that $F\beta$-Ψ-$cl(\delta \wedge \lambda) = \lambda$, $F\beta$-Ψ-$cl(\gamma \wedge \mu) = \mu$ and $F\beta$-Ψ-$cl(\delta) \tilde{q} F\beta$-$\Psi$-$cl(\gamma)$.

Proposition 3.1. Let (X, τ)be a fuzzy topological space and let $\Psi \in FO(X)$. Then the following are equivalent:

(i) (X, τ) is fuzzy β-Ψ- almost -β- normal,

(ii) For λ, μ are fuzzy β-Ψ- closed sets with $\lambda \tilde{q} \mu$ and λ is fuzzy β - Ψ-regular closed, there exist a fuzzy β-Ψ-open set δ such that $\mu = F\beta$-Ψ-$cl(\delta \wedge \mu)$ and $\lambda \tilde{q} F\beta$-Ψ-$cl(\delta)$,

(iii) For λ is fuzzy β - Ψ- closed, γ is fuzzy β - Ψ- regular open such that $\lambda \leq \gamma$, there exists a fuzzy β-Ψ-open set δ such that $\lambda = F\beta$-Ψ- cl $(\lambda \wedge \delta) \leq F\beta$-$\Psi$-$cl(\delta) \leq \gamma$.

*Proof :***(i) $\Rightarrow$ (ii)** Let λ, μ be fuzzy β-Ψ-closed sets with $\lambda \tilde{q} \mu$ and λ be fuzzy β-Ψ-regular closed. Since (X, τ) is fuzzy β-Ψ-almost β-normal, there exist fuzzy β-Ψ-open sets γ, δ such that $\lambda = F\beta$-Ψ-$cl(\gamma \wedge \lambda) \leq F\beta$-$\Psi$-$cl(\gamma)$, $\mu = F\beta$-Ψ-$cl(\delta \wedge \mu) \leq F\beta$-$\Psi$-$cl(\delta)$ and $F\beta$-Ψ-$cl(\gamma) \tilde{q} F\beta$-$\Psi$-$cl(\delta)$. This implies that $\lambda \tilde{q} F\beta$-Ψ-$cl(\delta)$. Hence (i)$\Rightarrow$(ii) is proved.

(ii)$\Rightarrow$(i) Let λ, $\mu \in F\beta\Psi C(X)$ with $\lambda \tilde{q} \mu$ and λ is fuzzyβ-Ψ-regular closed. By(ii), there exists a fuzzy β-Ψ- open set δ such that $\mu = F\beta$-Ψ-$cl(\delta \wedge \mu)$ and $\lambda \tilde{q} F\beta$-Ψ-$cl(\delta)$. Let $\gamma = F\beta$-Ψ-$int(\lambda)$. Hence $\lambda = F\beta$-Ψ-$cl(\gamma \wedge \lambda)$. Also $F\beta$-Ψ-$cl(\gamma) = \lambda$ and hence $F\beta$-Ψ-$cl(\gamma) \tilde{q} F\beta$-$\Psi$-$cl(\delta)$. Hence (ii)$\Rightarrow$(i) is proved.

(i)$\Rightarrow$(iii) Let λ be fuzzy β-Ψ-closed and γ be fuzzy β-Ψ- regular open. Let $\lambda \leq \gamma$. Since γ is fuzzy β-Ψ-regular open, $1_X - \gamma$ is fuzzy β-Ψ-regular closed. Since (X, τ) is fuzzy β-Ψ- almost β-normal, there exist open sets η and δ such that $1_X - \gamma = F\beta$-Ψ-cl $(\eta \wedge (1_X - \gamma)) \leq F\beta$-$\Psi$-$cl(\eta)$, $\lambda = F\beta$-Ψ-$cl(\delta \wedge \lambda) \leq F\beta$-$\Psi$-$cl(\delta)$ and $F\beta$-Ψ-$cl(\eta) \tilde{q} F\beta$-$\Psi$-$cl(\delta)$. This implies that $(1_X - \gamma) \tilde{q} F\beta$-$\Psi$-$cl(\delta)$. Hence $F\beta$ - Ψ- $cl(\delta) \leq \gamma$. Thus (i)$\Rightarrow$(iii) is proved.

(iii)$\Rightarrow$(ii) Let λ, μ be fuzzy β-Ψ-closed sets with $\lambda \tilde{q} \mu$, and let λ be fuzzy β-Ψ-regular closed. Then $\mu \leq (1_X - \lambda)$ and $1_X - \lambda$ is fuzzy β-Ψ- regular open. By(iii), there exist a fuzzy β -Ψ-open set δ such that $\mu = F\beta$-Ψ-$cl(\delta \wedge \mu) \leq F\beta$-$\Psi$-$cl(\delta) \leq (1_X - \lambda)$. This implies that $F\beta$-Ψ-$cl(\delta) \tilde{q} \lambda$. Hence (iii)$\Rightarrow$(ii) is proved.

Definition 3.8. Let (X, τ) be a fuzzy topological space and $\Psi \in FO(X)$. Then (X, τ) is said to be a fuzzy β-Ψ**-extremally disconnected space** if fuzzyβ-Ψ-closure of every fuzzy β-Ψ-open set is fuzzyβ-Ψ-open.

Equivalently, A fuzzy topological space (X, τ) is said to be fuzzy β-Ψ-extremally disconnected space if and only if for any two fuzzy β-Ψ-open sets λ, μ with $\lambda \tilde{q} \mu$ such that $F\beta$-Ψ-$cl(\lambda) \tilde{q} F\beta$-$\Psi$-$cl(\mu)$.

Proposition 3.2. Let (X, τ)be a fuzzy topological space and Ψ ∈ FO(X). If (X, τ) is fuzzy β-Ψ-extremely disconnected and fuzzy β-Ψ-almost β-normal space, then (X, τ) is fuzzy β-Ψ-almost normal.

*Proof :*Let (X, τ) be a fuzzy β-Ψ-extramally disconnected and fuzzy β-Ψ-almost β-normal space. Let λ, μ be fuzzy β-Ψ-closed sets of (X, τ) respectively where one of which is fuzzy β-Ψ-regular closed such that $\lambda \tilde{q} \mu$. Since (X,τ) is fuzzy β-Ψ- almost β-normal, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \tilde{q} \gamma$ such that Fβ-Ψ-$cl(\delta \wedge \lambda) = \lambda$, F$\beta$-Ψ-$cl(\gamma \wedge \mu) = \mu$ and Fβ-Ψ-$cl(\delta)$ $\tilde{q}$ Fβ-Ψ-$cl(\gamma)$. This implies that $\lambda \leq$ Fβ-Ψ-$cl(\delta)$ and $\mu \leq$ Fβ-Ψ-$cl(\gamma)$. Since (X, τ) is fuzzy β-Ψ-extremally disconnected space, Fβ-Ψ-$cl(\delta)$ and Fβ-Ψ-$cl(\gamma)$ are fuzzy β-Ψ-open sets of (X, τ) respectively. Hence (X, τ) is fuzzy β-Ψ-almost normal.

Definition3.9. Let (X, τ) be a fuzzy topological space and Ψ ∈ FO(X). Then (X, τ) is said to be a fuzzy β-Ψ-**almost regular space** if for every fuzzy β-Ψ-regular closed set λ and a fuzzy point x_t such that $x_t \tilde{q} \lambda$, there exist fuzzy β-Ψ- open sets δ, γ with $\delta \tilde{q} \gamma$ such that $x_t \leq \delta$ and $\lambda \leq \gamma$.

Definition 3.10. Let (X, τ) be a fuzzy topological space and Ψ ∈ FO(X). Then (X,τ) is called a **fuzzy** β**-Ψ-T**$_1$ (denoted by Fβ-Ψ-T$_1$) space if for any two fuzzy sets λ, $\mu \in I^X$ with $\lambda \tilde{q} \mu$, there exist γ, $\delta \in$ FβΨO(X) such that $\lambda \leq \gamma$, $\mu \tilde{q} \gamma$ and $\mu \leq \delta$, $\lambda \tilde{q} \delta$.

.Equivalently, (X, τ) is called a **fuzzy** β**-Ψ-T**$_1$(denoted by Fβ-Ψ-T$_1$) space if every fuzzy point in (X, τ) is fuzzy β-Ψ-closed.

Proposition 3.3. Let (X, τ)be a fuzzy topological space and Ψ ∈FO(X). If (X, τ) is a fuzzy β-Ψ-T$_1$space and fuzzy β-Ψ-almost β-normal space, then (X, τ) is fuzzy β-Ψ-almost regular.

Proof: Let λ be a fuzzy β-Ψ- regular closed set of (X, τ) and x_t be a fuzzy point such that $x_t \tilde{q} \lambda$. Since (X, τ) is a fuzzy β-Ψ-T$_1$ space, every fuzzy point in (X, τ) is fuzzy β-Ψ-closed in fuzzy β-Ψ-T$_1$ space and since (X,τ) is fuzzy β-Ψ-almost β-normal, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \tilde{q} \gamma$ such that $x_t \leq \delta$, Fβ-Ψ-$cl(\gamma \wedge \lambda) = \lambda$ and Fβ-Ψ-$cl(\delta)$ $\tilde{q}$ Fβ-Ψ-$cl(\gamma)$. Since $\lambda \leq$ Fβ-Ψ-$cl(\gamma)$ and Fβ-Ψ-$cl(\delta)$ $\tilde{q}$ Fβ-Ψ-$cl(\gamma)$, $\lambda \leq (1_X -$Fβ-Ψ-$cl(\delta))$. This implies that there exist fuzzy β-Ψ-open sets δ and $(1_X -$ Fβ-Ψ-$cl(\delta))$ with $\delta \tilde{q} (1_X -$ Fβ-Ψ-$cl(\delta))$ such that $x_t \leq \delta$ and $\lambda \leq (1_X -$ Fβ-Ψ-$cl(\delta))$. Hence (X, τ) is fuzzy β-Ψ-almost regular.

Definition 3.11. Let (X, τ) be a fuzzy topological space and Ψ ∈ FO(X). A fuzzy set $\lambda \in I^X$ in (X, τ) is called a fuzzy β-Ψ-**Q-neighborhood** of a fuzzy point $x_\lambda \in$ FP(X) iff there exists a fuzzy β-Ψ-open set μ such that $x_\lambda\ q\ \mu \leq \lambda$. If λ is a fuzzy β-Ψ- open set (resp.fuzzyβ-Ψ-closed set),then λ is called a fuzzy β-Ψ-open Q-neighborhood (resp. fuzzy β-Ψ-closed Q-neighborhood) ofx_λ.

Definition 3.12. Let (X, τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). A fuzzy point x_t is said to be a fuzzy β-Ψ-θ-**cluster point** of a fuzzy set λ iff for every fuzzy β-Ψ-open Q-neighborhood δ of x_t, Fβ-Ψ-*cl*(δ) is fuzzy quasi coincident with λ. The set of all fuzzy β-Ψ-θ-cluster points of λ is called the fuzzy β-Ψ-θ-**closure** of λ and denoted by Fβ-Ψ-$cl_\theta(\lambda)$.

Definition 3.13. Let (X, τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). The fuzzy β-Ψ-θ-**interior** of λ denoted as Fβ-Ψ-$int_\theta(\lambda)$ and is defined as

Fβ-Ψ-$int_\theta(\lambda) = 1_X -$ Fβ-Ψ-$cl_\theta(1_X - \lambda)$.

Definition3.14. Let (X, τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). A fuzzy set $\lambda \in I^X$ is said to be fuzzy β-Ψ-θ-**closed** (resp. fuzzy β-Ψ-θ-**open**) if λ = Fβ-Ψ-$cl_\theta(\lambda)$ (resp. λ =Fβ-Ψ-$int_\theta(\lambda)$).

Equivalently, A fuzzy set $\lambda \in I^X$ in (X, τ) is said to be fuzzy β-Ψ-θ-**open** if and only if for each fuzzy point $x_t \leq \lambda$, there exists a fuzzy β-Ψ-open set μ such that $x_t \leq \mu \leq$ Fβ-Ψ-cl(μ) $\leq \lambda$.

Definition 3.15. Let (X, τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). Then (X, τ) is said to be fuzzy β-Ψ-**weak**-θ-**normal** if for any two fuzzy β-Ψ-θ- closed sets λ, μ such that $\lambda\ \tilde{q}\ \mu$, there exist fuzzy β-Ψ-open sets δ, γ with $\delta\ \tilde{q}\ \gamma$such that $\lambda \leq \delta$ and $\mu \leq \gamma$.

Definition 3.16. Let (X, τ) be a fuzzy topological space and $\Psi \in$ F*O*(X). Then (X, τ) is said to be fuzzy β-Ψ-*k*-**normal** if for any two fuzzy β-Ψ-regular closed sets λ, μ with $\lambda\ \tilde{q}\ \mu$, there exist fuzzy β-Ψ-open sets δ, γ wi t h $\delta\ \tilde{q}\ \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$.

Proposition3.4. Let $\Psi \in$ F*O*(X). A fuzzy topological space (X, τ) is fuzzy β-Ψ-almost regular if and only if forevery fuzzy β-Ψ-open set λ, Fβ-Ψ-*int*(Fβ-Ψ-*cl*(λ)) is fuzzy β-Ψ-θ-open.

Proof: Let $\lambda \in$ F$\beta\Psi$O(X). If Fβ-Ψ-*int*(Fβ-Ψ-*cl*(λ))=1_X, then the proof is obvious. If Fβ-Ψ-*int*(Fβ-Ψ-*cl*(λ)) $\neq 1_X$ and if for any fuzzy point $x_t \leq$ Fβ-Ψ-*int*(Fβ-Ψ-*cl*(λ)) and a fuzzy β-Ψ-regular closed set (1_X–*F*β-Ψ-*int*(*F*β-Ψ-*cl*(λ))), then $x_t\ \tilde{q}$ (1_X–Fβ-Ψ-*int*(Fβ-Ψ-*cl*(λ))). Since (X, τ) is fuzzy β-Ψ-almost regular,there exist fuzzy β-Ψ-open sets δ, γ with $\delta\ \tilde{q}\ \gamma$ such that $x_t \leq \delta$ and (1_X–*F*β-Ψ-*int*(*F*β-Ψ-*cl*(λ))) $\leq \gamma$. It follows that Fβ-Ψ-*cl*(δ) $\tilde{q}\ \gamma$ and hence Fβ-Ψ-*cl*(δ) $\leq (1_X - \gamma) \leq$ Fβ-Ψ-*int*(Fβ-Ψ-*cl*(λ)). By Definition 3.14., it is clear that Fβ-Ψ-*int*(Fβ-Ψ-*cl*(λ)) is fuzzy β-Ψ-θ-open.

To prove the converse, let λ be a fuzzy β-Ψ-regular closed set of (X, τ) and x_t be a fuzzy point such that $x_t\ \tilde{q}\ \lambda$. Then $x_t \leq (1_X - \lambda)$. Since $1_X - \lambda$ is fuzzy β-Ψ-regular open, $(1_X - \lambda)$ = Fβ-Ψ-*int*(Fβ-Ψ-*cl*(1_X–λ)), which is fuzzy β-Ψ-θ-open. By Definition 3.14., there exists a fuzzy β-Ψ-open set μ with $x_t \leq \mu$

such that Fβ-Ψ-$cl(\mu) \leq (1_X - \lambda)$. Thus $\mu \,\tilde{q}\, (1_X - F\beta\text{-}\Psi\text{-}cl(\mu))$, $x_t \leq \mu$ and $\lambda \leq (1_X F\beta\text{-}\Psi\text{-}cl(\mu))$. Hence (X, τ) is fuzzy β-Ψ-almost regular.

Proposition 3.5. Let (X, τ) be a fuzzy topological space and let $\Psi \in FO(X)$. If (X, τ) is fuzzy β-Ψ-almost regular and fuzzy β-Ψ-weak-θ-normal, then (X, τ) is fuzzy β-Ψ-k-normal.

Proof: Assume (X, τ) is fuzzy β-Ψ-almost regular and fuzzy β-Ψ-weak-θ-normal. Let $\lambda, \mu \in F\beta\Psi RC(X)$ such that $\lambda \,\tilde{q}\, \mu$. Since λ is fuzzy β-Ψ-regular closed, $1_X-\lambda$ i s fuzzy β-Ψ-regular open and hence $1_X-\lambda = F\beta\text{-}\Psi\text{-}int(F\beta\text{-}\Psi\text{-}cl(1_X-\lambda))$. By Proposition 3.4., $1_X-\lambda$ is fuzzyβ-Ψ-θ-open and therefore λ is fuzzy β-Ψ-θ-closed. Similarly, μ is fuzzy β-Ψ-θ-closed. Since (X, τ) is fuzzy β-Ψ-weak-θ-normal, there exists fuzzy β-Ψ-open sets δ, γ with $\delta \,\tilde{q}\, \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$. Hence (X, τ) is fuzzy β-Ψ-k-normal.

Proposition 3.6. Let (X, τ) be a fuzzy topological space and let $\Psi \in FO(X)$. If (X, τ) is fuzzyβ-Ψ-T_1, fuzzy β-Ψ-weak-θ-normal and fuzzy β-Ψ-almost β-normal, then (X, τ) is fuzzyβ-Ψ-k-normal.

Proof: Let (X, τ) be fuzzy β-Ψ-T_1, fuzzy β-Ψ-weak-θ-normal and fuzzy β-Ψ-almost β-normal. By Proposition3.3., (X, τ) is fuzzy β-Ψ-almost regular. Again by Proposition 3.5., (X, τ) is fuzzy β-Ψ-k-normal.

Definition 3.17. Let (X, τ) be a fuzzy topological space and let $\Psi \in FO(X)$. Then (X, τ) is said to be fuzzy **β-Ψ-θ-normal** if for any two fuzzy β-Ψ-closed sets λ, μ one of which is fuzzy β-Ψ-θ-closed such that $\lambda \,\tilde{q}\, \mu$, there exist fuzzy$\beta$-$\Psi$-open sets δ, γ with $\delta \,\tilde{q}\, \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$.

Proposition 3.7. Let (X, τ) be a fuzzy topological space and let $\Psi \in FO(X)$. If (X, τ) is fuzzy β-Ψ-almost regular and fuzzy β-Ψ-θ-normal, then (X, τ) is fuzzy β-Ψ-almost normal.

Proof : Let (X, τ) be fuzzy β-Ψ-almost regular and fuzzy β-Ψ-θ-normal. Let λ, μ be fuzzy β-Ψ-closed sets of (X, τ) with $\lambda \,\tilde{q}\, \mu$ such that λ is fuzzy β -Ψ-regular closed. Hence $1_X-\lambda$ is fuzzy β-Ψ-regular open. This implies $1_X-\lambda = F\beta\text{-}\Psi\text{-}int(F\beta\text{-}\Psi\text{-}cl(1_X-\lambda))$. By Proposition 3.4., $1_X-\lambda$ is fuzzy β-Ψ-θ-open and hence λ is fuzzy β-Ψ-θ-closed. Since (X, τ) is fuzzyβ-Ψ-θ-normal, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \,\tilde{q}\, \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$. Hence (X, τ) is fuzzy β-Ψ-almost normal.

Proposition 3.8. Let (X, τ) be a fuzzy topological space and let $\Psi \in FO(X)$. If (X, τ) is fuzzy β-Ψ-T_1, fuzzy β-Ψ-θ-normal and fuzzy β-Ψ-almost β-normal, then (X, τ) is fuzzy β-Ψ-almost-normal.

Proof : Let (X, τ) be fuzzy β-Ψ-T_1, fuzzy β-Ψ-θ-normal and fuzzy β-Ψ-almost β-normal. By Proposition 3.3., (X, τ) is fuzzy β-Ψ-almost regular. Again by Proposition 3.7., (X, τ) is fuzzy β-Ψ-almost-normal.

FUZZY β-Ψ-SEMI-NORMAL SPACES

In this section, the notions of fuzzy β-Ψ-semi-normal spaces, fuzzy β-Ψ-regular spaces and fuzzyβ-Ψ-α-normal spaces are introduced and some of their properties are discussed.

Definition4.1. Let (X, τ) be a fuzzy topological space and $\Psi \in FO(X)$. Then (X, τ) is said to be fuzzy **β-Ψ-semi-normal** if for every fuzzy β-Ψ-closed set λ and a fuzzy β-Ψ-open set μ such that $\lambda \leq \mu$, there exists a fuzzy β-Ψ-regular open set γ such that $\lambda \leq \gamma \leq \mu$.

Definition 4.2. Let (X, τ) be a fuzzy topological space and $\Psi \in FO(X)$. Then (X, τ) is said to be fuzzy **β-Ψ-regular** if for every fuzzy point $x_t \in FP(X)$ and every fuzzy β-Ψ-closed set λ such that $x_t \tilde{q} \lambda$, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \tilde{q} \gamma$ such that $x_t \leq \delta$ and $\lambda \leq \gamma$.

Proposition 4.1. Let (X, τ) be a fuzzy topological space and let $\Psi \in FO(X)$. If (X, τ) is fuzzy β-Ψ-T_1, fuzzy β-Ψ-semi-normal and fuzzy β-Ψ-almost β-normal, then (X, τ) is fuzzy β-Ψ-regular.

*Proof :*Let (X, τ) be a fuzzy β-Ψ-T_1, fuzzy β-Ψ-semi-normal and fuzzy β-Ψ-almost β-normal space. Let λ be a fuzzy β-Ψ-closed set and a fuzzy point x_t such that $x_t \tilde{q} \lambda$. Since (X, τ) is fuzzy β-Ψ-T_1, every fuzzy point $x_t \in FP(X)$ is fuzzy β-Ψ-closed. Since (X, τ) is fuzzy β-Ψ-semi-normal, there exists a fuzzy β-Ψ-regular open set μ such that $x_t \leq \mu \leq 1_X - \lambda$. Now $\gamma = 1_X - \mu$ is a fuzzy β-Ψ- regular closed set such that $\lambda \leq \gamma$ with $x_t \tilde{q} \gamma$. Also since (X, τ) is fuzzy β-Ψ-T_1 and fuzzy β-Ψ-almost β-normal, by Proposition 3.3., (X, τ) is fuzzy β-Ψ-almost regular. This implies that there exist fuzzy β-Ψ-open sets δ, η with $\delta \tilde{q} \eta$ such that $x_t \leq \delta$ and $\lambda \leq \gamma \leq \eta$. Hence (X, τ) is fuzzyβ-Ψ-regular.

Proposition 4.2. Let (X, τ) be a fuzzy topological space and let $\Psi \in FO(X)$. If (X, τ) is fuzzy β-Ψ-almost β-normal and fuzzy β-Ψ-k-normal, then (X, τ) is fuzzy β-Ψ-almost normal.

*Proof :*Let (X, τ) be a fuzzy β-Ψ-almost β-normal and fuzzy β-Ψ-k-normal space. Let λ, μ be fuzzy β-Ψ-closed sets in which λ is a fuzzy β-Ψ-regular closed set such that $\lambda \tilde{q} \mu$. Since (X, τ) is fuzzy β-Ψ-almost β-normal, there exist fuzzy β-Ψ- open sets δ, γ with $\delta \tilde{q} \gamma$ such that $F\beta\text{-}\Psi\text{-}cl(\lambda \wedge \delta) = \lambda$ and $F\beta\text{-}\Psi\text{-}cl(\mu \wedge \gamma) = \mu$. This implies that $\lambda \leq F\beta\text{-}\Psi\text{-}cl(\delta)$ and $\mu \leq F\beta\text{-}\Psi\text{-}cl(\gamma)$. Hence $F\beta\text{-}\Psi\text{-}cl(\delta)$ and $F\beta\text{-}\Psi\text{-}cl(\gamma)$ are fuzzy β-Ψ-regular closed sets with $F\beta\text{-}\Psi\text{-}cl(\delta) \tilde{q} F\beta\text{-}\Psi\text{-}cl(\gamma)$. Since (X, τ)is fuzzy β-Ψ-k-normal, there exist fuzzy β-Ψ-open sets η_1, η_2 with $\eta_1 \tilde{q} \eta_2$ such that $F\beta\text{-}\Psi\text{-}cl(\delta) \leq \eta_1$ and $F\beta\text{-}\Psi\text{-}cl(\gamma) \leq \eta_2$. Hence (X, τ) is fuzzy β-Ψ-almost normal.

Definition 4.3. Let (X, τ) be a fuzzy topological space and $\Psi \in FO(X)$. Then (X, τ) is said to be fuzzy β-Ψ-α-**normal** if for anytwo fuzzy β-Ψ-closed sets λ, μ such that $\lambda \tilde{q} \mu$, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \tilde{q} \gamma$ such that $F\beta$-Ψ-$cl(\lambda \wedge \delta) = \lambda$ and $F\beta$-Ψ-$cl(\mu \wedge \gamma) = \mu$.

Proposition 4.3. Let (X, τ) be a fuzzy topological space and let $\Psi \in FO(X)$. If (X, τ) is a fuzzy β-Ψ-semi-normal and fuzzy β-Ψ-almost β-normal space, then (X, τ) is fuzzy β-Ψ-α-normal.

Proof : Let (X, τ) be a fuzzy β-Ψ-semi-normal and fuzzy β-Ψ-almost β-normal space. Let λ, μ be fuzzy β-Ψ-closed sets such that $\lambda \tilde{q} \mu$. This implies that $\lambda \leq 1_X - \mu$. Since (X, τ) is fuzzy β-Ψ-semi-normal, there exists a fuzzyβ-Ψ-regular open set γ such that $\lambda \leq \gamma \leq 1_X - \mu$. Now λ and $1_X - \gamma$ are fuzzy β-Ψ-closed sets in which $1_X - \gamma$ is a fuzzy β-Ψ-regular closed set such that $\mu \leq 1_X - \gamma$. Since (X, τ) is fuzzy β-Ψ-almost β-normal, there exist fuzzy β-Ψ-open sets δ, η with $\delta \tilde{q} \eta$ such that $F\beta$-Ψ-$cl(\lambda \wedge \delta) = \lambda$ and $F\beta$-Ψ-$cl((1_X - \gamma) \wedge \eta) = 1_X - \gamma$ and $F\beta$-Ψ-$cl(\delta) \tilde{q} F\beta$-$\Psi$- $cl(\eta)$. Here $\lambda = F\beta$-Ψ-$cl(\lambda \wedge \delta) \leq F\beta$-$\Psi$-$cl(\delta)$ and $1_X - \gamma = F\beta$-Ψ-$cl((1_X - \gamma) \wedge \eta) \leq F\beta$-$\Psi$-$cl(\eta)$. Hence δ and $1_X - (F\beta$-Ψ-$cl(\delta))$ are fuzzy β-Ψ-open sets with $\delta \tilde{q} 1_X - (F\beta$- Ψ-$cl(\delta))$ such that $F\beta$-Ψ-$cl(\delta \wedge \lambda) = \lambda$ and $\mu \leq 1_X - \gamma \leq 1_X - F\beta$-$\Psi$-$cl(\delta)$. Therefore $F\beta$-Ψ-$cl((1_X - F\beta$-Ψ-$cl(\delta)) \wedge \mu) = \mu$. Hence (X, τ) is fuzzy β-Ψ-α-normal.

FUZZY β-Ψ-NEARLY NORMAL SPACES

In this section, the concepts of fuzzy β-Ψ-nearly normal spaces, fuzzy β-Ψ–weak- θ-regular spaces are introduced and studied.

Definition 5.1. Let (X, τ) be a fuzzy topological space and $\Psi \in FO(X)$. Let λ be any fuzzy set. Then λ is said to be a **fuzzy** β-Ψ-δ-**closed set** if $\lambda = F\beta$-Ψ-$cl_\delta(\lambda)$ where $F\beta$-Ψ-$cl_\delta(\lambda) = \wedge\{\mu / \lambda \leq \mu$ *and* $\mu = F\beta$-Ψ-cl($F\beta$-Ψ-int(μ)) .The complement of a fuzzy β-Ψ-δ-closed set is fuzzyβ-Ψ-δ-open.

Definition 5.2. Let (X, τ) be a fuzzy topological space and $\Psi \in FO(X)$. Then (X, τ) is said to be a fuzzy β-Ψ-**nearly normal space** if for any two fuzzy β-Ψ-closed sets λ, μ with $\lambda \tilde{q} \mu$ one of which is fuzzy β-Ψ-δ- closed and the other is fuzzy β-Ψ- regular closed, there exist fuzzyβ-Ψ-open sets δ, γ with $\delta \tilde{q} \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$.

Remark 5.1. (i) Every fuzzy β-Ψ-δ closed set is fuzzy β-Ψ-closed.

(ii) Every fuzzy β-Ψ-regular-θ-closed set is fuzzy β-Ψ-regular closed.

Proposition 5.1. Let (X, τ) be a fuzzy topological space and $\Psi \in FO(X)$. If (X, τ) is a fuzzy β-Ψ-almost regular and fuzzy β-Ψ-θ-normal space, then (X, τ) is a fuzzy β-Ψ-nearly normal space.

Proof : Let (X, τ) be a fuzzy β-Ψ-almost regular and fuzzy β-Ψ-θ-normal space. Let λ, μ be fuzzy β-Ψ-closed sets such that $\lambda \tilde{q} \mu$ in which λ is fuzzy β-Ψ-regular closed and μ is fuzzy β-Ψ-δ-closed. Since every fuzzy β-Ψ-δ-closed set is fuzzy β-Ψ-closed, μ is fuzzy β-Ψ-closed. Since (X, τ) is fuzzy β-Ψ-almost regular, by Proposition3.4., λ is fuzzy β-Ψ-θ-closed. Since (X, τ) is fuzzy β-Ψ-θ-normal, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \tilde{q} \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$. Hence (X, τ) is fuzzy β-Ψ-nearly normal space.

Definition 5.3. Let (X, τ) be a fuzzy topological space and Ψ ∈ F*O*(X). Then (X, τ) is said to be a fuzzy β-Ψ**-weak-θ-regular space** if for every fuzzy β-Ψ-θ-closed set λ and a fuzzy β-Ψ-open set μ such that $\lambda \leq \mu$, there exists a fuzzy β-Ψ-θ-open set γ such that $\lambda \leq \gamma \leq \mu$.

Proposition 5.2. Let (X, τ) be a fuzzy topological space and Ψ ∈ F*O*(X). If (X, τ) is fuzzy β-Ψ-almost regular, fuzzy β-Ψ-weak-θ-regular and fuzzy β-Ψ-weak-θ- normal, then (X, τ) is fuzzy β-Ψ-nearly normal.

Proof: Let (X, τ) be a fuzzy β-Ψ-almost regular, fuzzy β-Ψ-weak-θ-regular and fuzzy β-Ψ-weak-θ-normal space. Let λ, μ be fuzzy β-Ψ-closed sets such that $\lambda \tilde{q} \mu$ in which λ is fuzzy β-Ψ-regular closed and μ is fuzzy β-Ψ-δ-closed. Since(X, τ) is fuzzy β-Ψ-almost regular, by Proposition 3.4., λ is fuzzy β-Ψ-θ-closed. Now $1_X-\mu$ is fuzzy β-Ψ-θ-open set such that $\lambda \leq 1_X-\mu$. Since (X, τ) is a fuzzy β-Ψ-weak-θ-regular space, there exists a fuzzy β-Ψ-θ-open set δ such that $\lambda \leq \delta \leq 1_X-\mu$. Hence $\mu \leq 1_X-\delta$ and $1_X-\delta$ is fuzzyβ-Ψ-θ-closed sets such that $\lambda \tilde{q} 1_X-\delta$. As (X, τ) is fuzzy β-Ψ-weak-θ-normal, there exist fuzzy β-Ψ-open sets γ, η with $\gamma \tilde{q} \eta$ such that $\lambda \leq \gamma$ and $1_X-\delta \leq \eta$. This implies that $\mu \leq \eta$. Hence (X, τ) is fuzzy β-Ψ-nearly normal.

FUZZY β-Ψ-WEAK k-NORMAL SPACES

In this section, the notion of fuzzy β-Ψ-weak k-normal spaces is introduced. Also an interesting characterisation is discussed.

Definition 6.1. Let (X, τ) be a fuzzy topological space and Ψ ∈ F*O*(X). A fuzzy β-Ψ-θ-closed set λ is said to be a fuzzy β-Ψ**-regular θ-closed** set if λ = Fβ- Ψ-*cl*(Fβ-Ψ-*int*(λ)).

Definition6.2. Let(X, τ) be a fuzzy topological space and Ψ ∈ F*O*(X). Then (X, τ) is said to be a fuzzy β-Ψ**-weak k-normal space** if for any two fuzzy β-Ψ-regular θ-closed sets λ, μ such that $\lambda \tilde{q} \mu$, there exist fuzzy β-Ψ-open sets δ, γ wi t h $\delta \tilde{q} \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$.

Proposition 6.1. Let (X, τ) be a fuzzy topological space and $\Psi \in FO(X)$. Then (X, τ) is fuzzyβ-Ψ-weak k-normal if and only if for every fuzzy β-Ψ-regular θ-closed set λ and a fuzzy β-Ψ-regular θ-open set μ such that $\lambda \leq \mu$, there exists a fuzzy β-Ψ-open set δ such that $\lambda \leq \delta \leq F\beta$-$\Psi$-$cl(\delta) \leq \mu$.

Proof : Let (X, τ) be a fuzzy β-Ψ-weak k-normal space. Let there be a fuzzy β- Ψ-regular θ-closed set λ and a fuzzy β-Ψ-regular θ-open set μ such that $\lambda \leq \mu$. Then λ and $1_X - \mu$ are fuzzy β-Ψ-regular θ-closed sets such that $\lambda \hat{q}\ 1_X - \mu$. Since (X, τ) is fuzzy β-Ψ-weak k-normal, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \tilde{q}\ \gamma$ such that $\lambda \leq \delta$ and $1_X - \mu \leq \gamma$. Therefore $\lambda \leq \delta \leq 1_X - \gamma \leq \mu$. Since $1_X - \gamma$ is fuzzy β-Ψ-closed, $\lambda \leq \delta \leq F\beta$-$\Psi$-$cl(\delta) \leq 1_X - \gamma \leq \mu$. This implies that $\lambda \leq \delta \leq F\beta$-$\Psi$-$cl(\delta) \leq \mu$.

Conversely assume that λ and μ be fuzzy β-Ψ-regular θ-closed sets in (X, τ) such that $\lambda \hat{q}\ \mu$. Then $1_X - \mu$ is fuzzy β-Ψ-regular θ-open set such that $\lambda \leq 1_X - \mu$. Thus by hypothesis, there exists a fuzzy β-Ψ-open set δ such that $\lambda \leq \delta \leq F\beta$-$\Psi$-$cl(\delta) \leq 1_X - \mu$. Then δ and $1_X - F\beta$-Ψ-$cl(\delta)$ are fuzzy β-Ψ-open sets such that $\delta \hat{q}\ 1_X - F\beta$-$\Psi$-$cl(\delta)$ such that $\lambda \leq \delta$ and $\mu \leq 1_X - F\beta$-Ψ-$cl(\delta)$. Hence(X, τ) is fuzzy β-Ψ-weak k-normal.

Proposition 6.2. Let (X, τ) be a fuzzy topological space and $\Psi \in FO(X)$. For a fuzzy β-Ψ-almost regular space, the following are equivalent.

(i) (X, τ) is fuzzy β-Ψ-k-normal,

(ii) (X, τ) is fuzzy β-Ψ-weak-k-normal.

Proof : **(i) ⇒ (ii)** Let λ, μ be fuzzy β-Ψ-regular θ-closed sets such that $\lambda \hat{q}\ \mu$. Since every fuzzy β-Ψ-regular θ-closed set is fuzzy β-Ψ-regular closed, and since (X, τ) is fuzzy β-Ψ-k-normal, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \tilde{q}\ \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$. Hence (X, τ) is fuzzy β-Ψ-weak-k-normal.

(ii) ⇒ (i) Let (X, τ) be a fuzzy β-Ψ- almost regular and fuzzy β-Ψ-weak-k-normal space. Let λ, μ be fuzzy β-Ψ-regular closed sets in (X, τ) such that $\lambda \tilde{q}\ \mu$. By Proposition3.4., λ and μ are fuzzy β-Ψ-regular θ-closed sets such that $\lambda \tilde{q}\ \mu$. Since (X, τ) is fuzzy β-Ψ-weak-k-normal, there exist fuzzy β-Ψ-open sets δ, γ with $\delta \tilde{q}\ \gamma$ such that $\lambda \leq \delta$ and $\mu \leq \gamma$. Hence (X, τ) is fuzzy β-Ψ-k-normal.

CONCLUSION

The role of normality is vital in general topology. In this paper, several types of fuzzy normality's are studied via fuzzy operation. The results in this paper motivate to study further applications of fuzzy β-Ψ-almost β-normal spaces and other fuzzy β-Ψ-normal spaces.

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