Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Solution to Transportation problem in fuzzy environment with New Ranking Technique

Purushothkumar M.K. [1*], Ananthanarayanan M. [2] and Dhanasekar S. [3]

[1*](Department of Mathematics, Dr. Ambedkar Govt Arts college,Chennai-600039,Tamilnadu)
[2](Department of Mathematics, A.M. Jain College,Chennai-600114,Tamilnadu)
[3](Mathematics Division, SAS, VIT Chennai)

ABSTRACT

In this paper, a new ranking technique based on centroid ranking technique introduced for ordering of fuzzy numbers. First, we transform the fuzzy quantities as the cost, coefficients, supply and demands, into crisp quantities by using proposed ranking method and then by using the VAM algorithm to solve and obtain the initial basic feasible solution of the problem and optimal solution is obtained by Modified Distribution Method. Examples are furnished to validate the method.

KEYWORDS: Fuzzy set, Fuzzy transportation problem, Triangular Fuzzy number, Trapezoidal Fuzzy number, Ranking Technique**,** Vogel’s Approximation method, MODI method.

***Corresponding Author**

M.K.Purushothkumar

Assistant professor of Maths,

Dr.Ambedkar Govt.Arts College,Chennai-600039

Email: mkpurushoth2004@gmail.com

INTRODUCTION

A Transportation problem is to find the shipping schedule that minimizes the transportation cost. It was first developed by Hitchcock[1]. In real time examples an uncertainty is involved in fixing the decision variables such as transportation cost, supply and demand. , Zadeh[2] introduced the notation of fuzziness and it was restated by Bellman and Zadeh[3]. Zimmermann [4,5] helps to overcome this difficult. Chanas et al [6] proposed Parametric programming Technique to solve Fuzzy transportation problem. This method not only identifies the solution, but also all other alternatives. Chanas and Kuchta[7] converted the given problem into a bicriterial TP with a crisp objective function and solved. Liu Kao [8] used Extension principle to solve fuzzy transportation problem Verma et al [9] solved fuzzy transportation problem with hyperbolic and exponential membership function by appling the fuzzy programming technique. T.F.Liang et al [10] used fuzzy Linear programming to solve interactive Multi objective transportation planning decision problems.. Nagoor Gani and K. Abdul Razak [11] have solved fuzzy transportation problem in two stages. P. Pandian and G. Natrajan [12] has solved fuzzy transportation problem of trapezoidal numbers by introducing zero point method. Defuzzification is a process that converts a fuzzy set or fuzzy number into a crisp value or number. In 1981 R.R. Yager [13] procedure for ordering fuzzy subsets of the unit interval, S.H. Chen[14] Ranking fuzzy numbers with maximizing set and minimizing set. On the centorids of fuzzy numbers by Wang[15]. P. Fortemps and M. Roubens[16] introduced a ranking and defuzzification methods based on area compensation.

S. Abbasbandy and T. Hajjari[17] gave new approach for ranking of trapezoidal Fuzzy numbers . C.H.Cheng[18] developed a ranking technique by using distance method. A new method on ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviations by Chen and Chen [19] was derived. This paper is organized as follows: In section 2 some basic definitions which is required for our study are furnished. In section 3 new ranking function is proposed. In section 4 the proposed method is discussed and numerical examples are given. In section 5 deals with the conclusion.

PRELIMINARIES

2.1 Definition A fuzzy set $\tilde{A}$ of a universal set U is defined by a membership function $f_{\tilde{A}}: U \rightarrow [0,1]$,

2.2 Definition

A fuzzy number is a convex fuzzy subset of the real line R and is completely defined by its membership function. Let $\tilde{A}$ be a fuzzy number, whose membership function $f_{\tilde{A}}(x)$ can be defined as [4]

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}^{L}(x) & \text{if } a_1 \le x \le a_2 \\ \omega & \text{if } a_2 \le x \le a_3 \\ f_{\tilde{A}}^{R}(x) & \text{if } a_3 \le x \le a_4 \\ 0 & \text{othewise} \end{cases}$$

Where $0 < \omega \le 1$ is a constant, $f_{\tilde{A}}^{L} : [a_1, a_2] \to [0, \omega]$ and $f_{\tilde{A}}^{R} : [a_3, a_4] \to [0, \omega]$ are two strictly monotonically and continuous mapping R to closed interval $[0, \omega]$. If $\omega = 1$,then $\tilde{A}$ is a normal fuzzy number; otherwise it is said to be a non normal fuzzy number. If the membership function $f_{\tilde{A}}(x)$ is piecewise linear, then $\tilde{A}$ is referred to as a trapezoidal fuzzy number and is usually denoted by $\tilde{A} = (a_1, a_2, a_3, a_4; \omega)$ which is plotted in Fig 1.In particular, when $a_2 = a_3$,the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by $\tilde{A} = (a_1, a_3, a_4; \omega)$.

So, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers.

Since $f_{\tilde{A}}^{L}(x)$ and $f_{\tilde{A}}^{R}(x)$ are both strictly monotonically and continuous functions, their inverse functions exists and should also be continuous and strictly monotonical .Let $g_{\tilde{A}}^{L} : [0, \omega] \to [a_1, a_2]$ and $g_{\tilde{A}}^{R} : [0, \omega] \to [a_3, a_4]$ be the inverse functions of $f_{\tilde{A}}^{L}(x)$ and $f_{\tilde{A}}^{R}(x)$ respectively. Then $g_{\tilde{A}}^{L}(y)$ and $g_{\tilde{A}}^{R}(y)$ should be integrable on the closed interval $[0, \omega]$.In other words ,both $\int_0^{\omega} g_{\tilde{A}}^{L}(y)dy$ and

$\int_0^{\omega} g_{\tilde{A}}^{R}(y)dy$ should exists. In the case of trapezoidal fuzzy number the inverse functions $g_{\tilde{A}}^{L}(y)$ and $g_{\tilde{A}}^{R}(y)$ can be analytically expressed as

$$g_{\tilde{A}}^{L}(y) = a_1 + \frac{(a_2 - a_1)y}{\omega} \quad ,0 \le y \le \omega$$

$$g_{\tilde{A}}^{R}(y) = a_4 - \frac{(a_4 - a_3)y}{\omega}, \; 0 \le y \le \omega$$

Consider a generalised fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; \omega)$ whose membership function is defined as

$$f_{\tilde{A}}(x) = \begin{cases} \dfrac{\omega(x - a_1)}{(a_2 - a_1)} & \text{if } a_1 \le x \le a_2 \\ \omega & \text{if } a_2 \le x \le a_3 \\ \dfrac{\omega(a_4 - x)}{(a_4 - a_3)} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{othewise} \end{cases}$$

In order to determine centriod point $(\tilde{x}_0(\tilde{A}), \tilde{y}_0(\tilde{A}))$ of a fuzzy number $\tilde{A}$,and Wang[15] provided following centroid formulae:

$$\tilde{x}_0(\tilde{A}) = \frac{\int_{-\infty}^{\infty} x f_{\tilde{A}}(x)dx}{\int_{-\infty}^{\infty} f_{\tilde{A}}(x)dx}$$

$$= \frac{\int_{a_1}^{a_2} x f_{\tilde{A}}^{L}(x)dx + \int_{a_2}^{a_3} x\omega dx + \int_{a_3}^{a_4} x f_{\tilde{A}}^{R}(x)dx}{\int_{a_1}^{a_2} f_{\tilde{A}}^{L}(x)dx + \int_{a_2}^{a_3} \omega dx + \int_{a_3}^{a_4} f_{\tilde{A}}^{R}(x)dx}$$

$$= \frac{\int_{a_1}^{a_2} x \frac{\omega(x - a_1)}{(a_2 - a_1)} dx + \int_{a_2}^{a_3} x\omega dx + \int_{a_3}^{a_4} x \frac{\omega(a_4 - x)}{(a_4 - a_3)} dx}{\int_{a_1}^{a_2} \frac{\omega(x - a_1)}{(a_2 - a_1)} dx + \int_{a_2}^{a_3} \omega dx + \int_{a_3}^{a_4} \frac{\omega(a_4 - x)}{(a_4 - a_3)} dx}$$

$$\tilde{x}_0(\tilde{A}) = \frac{1}{3}\left[(a_1 + a_2 + a_3 + a_4) - \frac{(a_4 a_3 - a_1 a_2)}{(a_4 + a_3) - (a_1 + a_2)} \right]$$

$$\tilde{y}_0(\tilde{A}) = \frac{\int_0^{\omega} y[g_{\tilde{A}}^{R}(y) - g_{\tilde{A}}^{L}(y)]dy}{\int_0^{\omega} [g_{\tilde{A}}^{R}(y) - g_{\tilde{A}}^{L}(y)]dy}$$

$$= \frac{\int_0^{\omega} y\left([a_4 - (a_4 - a_3)\frac{y}{\omega}] - [a_1 + (a_2 - a_1)\frac{y}{\omega}] \right)dy}{\int_0^{\omega} \left([a_4 - (a_4 - a_3)\frac{y}{\omega}] - [a_1 + (a_2 - a_1)\frac{y}{\omega}] \right)dy}$$

$$\tilde{y}_0(\tilde{A}) = \frac{\omega}{3}\left[1 + \frac{a_3 - a_2}{(a_4 + a_3) - (a_1 + a_2)}\right]$$

Where $\tilde{x}_0(\tilde{A})$ and $\tilde{y}_0(\tilde{A})$ is the centorid of the general trapezoidal fuzzy number

Suppose Triangular Fuzzy number $\tilde{A} = (a_1, a_3, a_4; \omega)$ then

$$\tilde{x}_0(\tilde{A}) = \frac{1}{3}[(a_1 + a_3 + a_4)]$$

$$\tilde{y}_0(\tilde{A}) = \frac{\omega}{3}$$

2.3 Properties of Triangular and Trapezoidal fuzzy numbers.

Let $\tilde{A} = (a_1, a_2, a_3)$, $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then the fuzzy numbers addition fuzzy numbers ,subtraction and fuzzy members multiplication are defined as follows.

(i) $\tilde{A} + \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(ii) $\tilde{A} - \tilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

(iii) $\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3) \otimes (b_1, b_2, b_3) = (a_1 \otimes b\ , a_2 \otimes b_2, a_3 \otimes b_3)$

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers, then the fuzzy numbers addition fuzzy numbers ,subtraction and fuzzy members multiplication are defined as follows

(iv) $\tilde{A} + \tilde{B} = (a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(v) $\tilde{A} - \tilde{B} = (a_1, a_2, a_3, a_4) - (b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

(vi) $\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (t_1, t_2, t_3, t_4)$

Where $t_1 = \min(a_1 b_1, a_1 b_4, a_4 b_1, a_4 b_4)$

$t_2 = \min(a_2 b_2, a_2 b_3, a_3 b_2, a_3 b_3)$

$t_2 = \max(a_2 b_2, a_2 b_3, a_3 b_2, a_3 b_3)$

$t_1 = \max(a_1 b_1, a_1 b_4, a_4 b_1, a_4 b_4)$

I. Proposed Ranking Method

An efficient approach for comparing the fuzzy numbers is by use of a ranking function $R : F(R) \to R,$ where $F(R)$ is a fuzzy numbers defined on set of real numbers, which maps each fuzzy number into a real number, where natural order exists. Wang [15] used a centroid based distance approach to rank fuzzy numbers.

For trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; \omega)$, the ranking function is defined as

$$\Re(\tilde{A}) = \sqrt{\widetilde{x_0}^2(\tilde{A}) + \widetilde{y_0}^2(\tilde{A})}$$

Where $\tilde{x}_0(\tilde{A}) = \frac{1}{3}\left[(a_1 + a_2 + a_3 + a_4) - \frac{(a_4 a_3 - a_1 a_2)}{(a_4 + a_3) - (a_1 + a_2)}\right]$

$$\tilde{y}_0(\tilde{A}) = \frac{\omega}{3}\left[1 + \frac{a_3 - a_2}{(a_4 + a_3) - (a_1 + a_2)}\right].$$

For any two trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$ then we have

(i) $\tilde{A} \leq \tilde{B} \Leftrightarrow \Re(\tilde{A}) \leq \Re(\tilde{B})$

(ii) $\tilde{A} \geq \tilde{B} \Leftrightarrow \Re(\tilde{A}) \geq \Re(\tilde{B})$

(iii) $\tilde{A} = \tilde{B} \Leftrightarrow \Re(\tilde{A}) = \Re(\tilde{B})$

II. Mathematical Formulation Of Fuzzy Transformation Problem

The fuzzy transportation problems, in which a decision maker is uncertain about the precise value of transportation cost, availability and demand, can be formulated as follows

minimize $\tilde{z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$

Subject to $\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i, \quad i = 1,2,3,...,m.$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, \quad j = 1,2,3,...,n.$$

$$\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j, \quad i = 1,2,3,...,n,\ j = 1,2,3,...,n \quad \text{and} \quad \tilde{x}_{ij} \geq 0.$$

Where m = total number of sources

n = total number of destinations

$\tilde{a}_i$ = the fuzzy availability of the product at ith source

$\tilde{b}_i$ = the fuzzy demand of the product at jth destination

$\tilde{c}_{ij}$ = the fuzzy transportation cost for unit quantity of the product from i th source to j th destination

$\tilde{x}_{ij}$ = the fuzzy quantity of the product that should be transported from ith source to jth destination to minimize the total fuzzy transportation cost

$\sum_{i=1}^{m} \tilde{a}_i$ = total fuzzy availability of the product

$\sum_{j=1}^{n} \tilde{b}_j$ = total fuzzy demand of the product

$\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$ = total fuzzy transportation cost

If $\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.

4.1. Algorithm for Vogel Approximation method

Step 1. Convert the given fuzzy parameters in to crisp values by using proposed ranking method.

Step 2. If it is unbalanced convert the given fuzzy transportation problem to balanced transportation problem.

Step 3. Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next cell cost in the same row or column.

Step 4. Select the row or column with the highest penalty cost (breaking tiles arbitrarily or choosing the lowest cost cell).

Step 5. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.

Step 6. Repeat 3 and 4 until all requirements have been meet.

Step 7. Apply MODI method to get optimal solution.

4.2 Numerical Examples

EXAMPLE1. Consider the fuzzy transportation problem in the following table gives all the necessary information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost (in Rs) from each warehouse to each market. Here cost value, supplies and demands are triangular fuzzy numbers and FAi and FRi are fuzzy supply and fuzzy demand. The given problem is balanced transportation problem. The fuzzy initial basic feasible solution is obtained by .

Table 1: Numerical Example 1

	FR1	FR2	FR3	FR4	Fuzzy Supply
FA1	(1,5,9)	(4,9,14)	(9, 13,17)	(1,2,3)	(20,50,80)
FA2	(9,11,13)	(9,18,27)	(18,20,22)	(1,3,5)	(25,50,75)
FA3	(8,14,20)	(10,15,20)	(10,16,22)	(2,7,12)	(30,50,70)
Fuzzy Demand	(10,30,50)	(20,40,60)	(35,55,75)	(10,25,40)	(75,150,225)

By using new ranking method of the triangular fuzzy numbers,

$$\Re(\tilde{A}) = \sqrt{\widetilde{x_0}^2(\tilde{A}) + \widetilde{y_0}^2(\tilde{A})}$$

Where $\tilde{x}_0(\tilde{A}) = \frac{1}{3}[(a_1 + a_3 + a_4)]$ and $\tilde{y}_0(\tilde{A}) = \frac{\omega}{3}$

For taking $\omega = 1$,we have

$\Re(1, 5,9) = 5.01$ $\Re(9,11,13) = 11.01$ $\Re(8,14,20) = 14$

$\Re(4,9,14) = 9.01$ $\Re(9,18,27) = 18$ $\Re(10,15,20) = 15$

$\Re(9,13,17) = 13$ $\Re(18,20,22) = 20$ $\Re(10,16,22) = 16$

$\Re(1,2,3) = 2.03$ $\Re(1,3,5) =3.02$ $\Re(2,7,12) = 7.01$

Rank of all Supply: $\Re(20,50,80) = 50$, $\Re(25,50,75) = 50$, $\Re(30,50,70) = 50$

Rank of all fuzzy Demand: $\Re(10,30,50) = 30$, $\Re(20,40,60) = 40$, $\Re(35,55,75) = 55$,

$\Re(10,25,40) = 25$.

Substitute these values in fuzzy transportation problem, we get the crisp transportation problem which is shown following table.

Table 2: Tranportation Table -1

	FR1	FR2	FR3	FR4	Fuzzy Supply
FA1	5.01	9.01	13	2.03	50
FA2	11.01	18	20	3.02	50
FA3	14	15	16	7.01	50
Fuzzy Demand	30	40	55	25	150

The fuzzy transportation problem is balanced. After applying the VAM procedure for Initial Basic Feasible solution, the allocations are as follows

Minimum Transportation cost = (5.01 X 5) + (9.01 X 40)+(13 X 5) + (11.01 X 25) +

(3.02 X 25)+ (16 X 50)

=1601.2

Using MODI method, the optimal solution is given by

Table 3: Transportation Table-2

	FR1	FR2	FR3	FR4	
FA1	$\boxed{5}$ 5.01	$\boxed{40}$ 9.01	$\boxed{5}$ 13	-2.98 2.03 5.01	$u_1 = 0$
FA2	$\boxed{25}$ 11.01	15.01 18 2.99	19 20 1	$\boxed{25}$ 3.02	$u_2 = 6$
FA3	8.01 14 5.99	12.01 15 2.99	$\boxed{50}$ 16	0.02 7.01 6.99	$u_3 = 3$

$v_1 = 5.01$ $v_2 = 9.01$ $v_3 = 13$ $v_4 = -2.98$

The crisp value of the fuzzy transportation problem is:

Total cost = (5.01 X 5) + (9.01 X 40)+(13 X 5) + (11.01 X 25) +(3.02 X 25)+ (16 X 50)

=1601.2

EXAMPLE2.

Consider the fuzzy transportation problem in the following table gives all the necessary information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost (in Rs) from each warehouse to each market. Here cost value, supplies and demands are trapezoidal fuzzy numbers and FAi and FRi are fuzzy supply and fuzzy demand. The given problem is balanced transportation problem. The fuzzy initial basic feasible solution is obtained by .

Table 4: Numerical Example 2

	FR1	FR2	FR3	FR4	Fuzzy Supply
FA1	(1, 2, 3, 4)	(1, 3, 4, 6)	(9, 11, 12, 14)	(5, 7, 8, 11)	(1, 6, 7, 12)
FA2	(0, 1, 2, 4)	(–1, 0, 1, 2)	(5, 6, 7, 8)	(0, 1, 2, 3)	(0, 1, 2, 3)
FA3	(3, 5, 6, 8)	(5, 8, 9, 12)	(12, 15, 16, 19)	(7, 9, 10, 12)	(5, 10, 12, 17)
Fuzzy Demand	(5, 7, 8, 10)	(1, 5, 6, 10)	(1, 3, 4, 6)	(1, 2, 3, 4)	

By using new ranking method of the trapezoidal fuzzy numbers,

$$\Re(\tilde{A}) = \sqrt{\widetilde{x_0}^2(\tilde{A}) + \widetilde{y_0}^2(\tilde{A})}$$

Where $\tilde{x}_0(\tilde{A}) = \frac{1}{3}\left[(a_1+a_2+a_3+a_4) - \frac{(a_4a_3 - a_1a_2)}{(a_4+a_3)-(a_1+a_2)}\right]$

$$\tilde{y}_0(\tilde{A}) = \frac{\omega}{3}\left[1 + \frac{a_3 - a_2}{(a_4+a_3)-(a_1+a_2)}\right].$$

For taking $\omega = 1$, we have

$\Re(1, 2, 3, 4) = 2.54$ $\Re(0, 1, 2, 4) = 1.84$ $\Re(3, 5, 6, 8) = 5.51$

$\Re(1, 3, 4, 6) = 3.52$ $\Re(-1, 0, 1, 2) = 0.65$ $\Re(5, 8, 9, 12) = 8.51$

$\Re(9, 11, 12, 14) = 11.51$ $\Re(5, 6, 7, 8) = 6.51$ $\Re(12, 15, 16, 19) = 15.51$

$\Re(5, 7, 8, 11) = 7.82$ $\Re(0, 1, 2, 3) = 1.56$ $\Re(7, 9, 10, 12) = 9.51$

Rank of all Supply: $\Re(1, 6, 7, 12) = 6.51$, $\Re(0, 1, 2, 3) = 1.56$, $\Re(5, 10, 12, 17) = 11.01$

Rank of all fuzzy Demand: $\Re(5, 7, 8, 10) = 7.51$, $\Re(1, 5, 6, 10) = 5.51$, $\Re(1, 3, 4, 6) = 3.52$,

$\Re(1, 2, 3, 4) = 2.54$.

Substitute these values in fuzzy transportation problem; we get the crisp transportation problem which is shown following table.

Table 5: Transportation Table-3

	FR1	**FR2**	**FR3**	**FR4**	**Fuzzy Supply**
FA1	2.54	3.52	11.51	7.82	6.51
FA2	1.84	0.65	6.51	1.56	1.56
FA3	5.51	8.51	15.51	9.51	11.01
Fuzzy Demand	7.51	5.51	3.52	2.54	19.08

The fuzzy transportation problem is balanced. After applying the VAM procedure for Initial Basic Feasible solution, the allocations are as follows

Table 6: Transportation Table-4

	FR1	**FR2**	**FR3**	**FR4**	**Fuzzy Supply**
FA1	$\boxed{1}$ 2.54	$\boxed{5.51}$ 3.52	11.51	7.82	6.51
FA2	1.84	0.65	6.51	$\boxed{1.56}$ 1.56	1.56
FA3	$\boxed{6.51}$ 5.51	8.51	$\boxed{3.52}$ 15.51	$\boxed{0.98}$ 9.51	11.01
Fuzzy Demand	7.51	5.51	3.52	2.54	19.08

.

Minimum Transportation cost = $(2.54 \times 1) + (3.52 \times 5.51) + (1.56 \times 1.56) + (5.51 \times 6.51) + (15.51 \times 3.52) + (9.51 \times 0.98) = 124.1539.$

which is not optimal solution.

Using MODI method, the optimal solution is given by

Table7: Transportation Table-5

	FR1	**FR2**	**FR3**	**FR4**
FA1	2.54	$\boxed{5.51}$ 3.52	$\boxed{1}$ 11.51	7.82
FA2	1.84	0.65	$\boxed{1.56}$ 6.51	1.56
FA3	$\boxed{7.51}$ 5.51	8.51	$\boxed{0.96}$ 15.51	$\boxed{2.54}$ 9.51

The above table satisfies the rim conditions with (m+n–1) non negative allocations at independent positions.

Thus the optimal allocation is

$x_{12} = 5.51, \quad x_{13} = 1, \quad x_{23} = 1.56, \quad x_{31} = 7.51, \quad x_{33} = 0.96, \quad x_{34} = 2.54$

The crisp value of the fuzzy transportation problem is:

Total cost = $(3.52 \times 5.51) + (11.51 \times 1) + (6.51 \times 1.56) + (5.51 \times 7.51) + (15.51 \times 0.96) + (9.51 \times 2.54)$

$= 121.4859.$

III. Conclusion

In this paper, an effective ordering of fuzzy numbers is introduced and applied for solving fuzzy transportation problem. More over fuzzy transportation problem has been transformed into crisp transportation problem using ranking method. It is easy to understand and compute since it follows the step of crisp transportation problem. Numerical examples validate the effectiveness of the proposed method.

REFERENCES

1. Hictchcock FL. The distribution of a product from several sources to numerous localities. Journal of Mathematical Physics, 1941; 224–230.
2. Zadeh LA. Fuzzy sets, information and control, , 1965; 8: 338–353
3. Bellman RE, Zadeh LA. Decision making in a fuzzy environment. Management Sci. 1970; 17,141–164.
4. Zimmermann HJ. Fuzzy set theory and its applications. Fourth Edition ISBN 0792374355, 1934.
5. Zimmermann HJ. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems. 1978; 45–55.
6. S.Chanas,W.Kolodziejczyk, and A.Macharaj , A Fuzzy approach to the transportation problem , Fuzzy Sets and Systems 1984; 13: 211-224.
7. Chanas S, Kuchta D. A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Fuzzy Sets and Systems, 1996; 82: 299–305.
8. Shiang-Tai Liu and Chiang Kao. Solving fuzzy transportation problems based on extension principle , European Journal of Operational Research, 2004; 153: 661-674.
9. R.Verma,M.P.Biswal and A.Biswas. Fuzzy programming technique to solve multi objective Transportation problems with some non-linear membership functions, Fuzzy Sets and Systems, 1997; 91: 37-43.
10. T.F.Liang. Interactive Multi objective Transportation planning decisions using fuzzy linear programming , Asia-Pacific Journal of operation research , 2008; 25(01): 11-31
11. Gani A, Razak KA. Two stage fuzzy transportation problem. JournaSl of Physical Sciences, 2006; 63–69.
12. Pandian P, Natarajan G. A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem. Applied Mathematical Science, 2010; 4: 79–90.

13. Yager R.R. “A procedure for ordering fuzzy subsets of the unit interval”, Information Sciences, 1981; 24: 143–161
14. Chen, S. H. Ranking fuzzy number with maximizing set and minimizing set, Fuzzy Sets and Systems, 1985;17(1): 113–129.
15. Ying-Ming Wang,Jian-Bo Yang,Dong-Ling Xu,Kwai-Sang Chin .On the centroids of fuzzy number .Fuzzy set and systems, 2006; 919-926
16. Fortemps, P. and Roubens, M.. Ranking and defuzzification methods based on area compensation, Fuzzy Sets and Systems, 1996; 82: 319–330.
17. Abbasbandy, S. and Hajjari, T. A new approach for ranking of trapezoidal fuzzy numbers. omputers and Mathematics with Applications, 2009; 57(3): 413–419.
18. Chen SJ, Chen SM. Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers. Applied Intelligence, 2007 ; 26: 1–11.
19. C.H.Chen .A new approach for ranking fuzzy numbers by distance method,Fuzzy sets and systems, 1998; 95: 307-317.

.