Research article Available online www.ijsrr.org ISSN: 2279–0543

International Journal of Scientific Research and Reviews

Dynamical Interactions in an Initially Stressed Micropolar Thermo Diffusive Medium with Fractional order Heat Conduction

Jain Kavita[1*] and Yadav Renu[2]

[1]Department of Mathematics, Guru Jambheshwar University of Science and Technology, Hisar 125001, India

[2]Department of Mathematics, Government College, Adampur, Hisar, Haryana, India

ABSTRACT

The objective of this paper is to study dynamical deformations in an initially stressed diffusive micropolar thermoelastic medium with fractional order heat conduction subjected to ramp type mechanical load. The theory of fractional order generalized thermoelasticity is employed in an initially stressed micropolar thermoelastic half space with diffusion. Laplace and Fourier transforms are employed to solve the problem. Expressions for different field variables in the physical domain are derived by the application of numerical inversion technique. Some particular cases of interest have also been inferred from the present problem. Comparisons of the physical quantities are shown in figures to study the effects of ramp parameter, fractional parameter and initial stress.

KEYWORDS: Fractional order thermoelasticity, micropolar, diffusion, initial stress.

*Corresponding author

Kavita Jain

Department of Mathematics,

Guru Jambheshwar University of Science and Technology,

Hisar, Haryana, India.

Email- kavitaj217@gmail.com

INTRODUCTION

Eringen's micropolar theory of elasticity[1] is now well known and does not need much introduction. A historical development of the theory of micropolar elasticity is given in a monograph by Eringen[2]. In this theory, a load across a surface element is transmitted not only by a force stress vector but also by a couple stress vector and the motion is characterized by six degrees of freedom (three of translation and three of microrotation). Micropolar elastic solids can be thought of as being composed of dumb-bell type molecules and these molecules in a volume element can undergo rotation about their centre of mass along with the linear displacement. This theory is expected to find applications in the treatment of mechanics of granular materials, composite fibrous materials and particularly microcracks and microfractures. The dynamical interactions between thermal and mechanical fields in solids have great applications in aeronautics, nuclear reactors and high energy particle accelerators. Keeping the above applications in view, the micropolar theory was extended to include thermal effects by Nowacki[3-5] and Eringen[6]. One can refer to Dhaliwal and Singh[7] for a review on the micropolar thermoelasticity and a historical survey of the subject.

In the last few years, fractional calculus has also been introduced in the theory of thermoelasticity. A quasi-static uncoupled theory of thermoelasticity based on fractional heat conduction equation has been put forward by Povstenko[8]. Sherief *et al.*[9] have proposed a new model of thermoelasticity using fractional calculus with second sound effects, proved a uniqueness theorem and derived reciprocity relation and variational principle. Shaw and Mukhopadhyay[10] have suggested a fractional order micropolar generalized thermoelasticity theory with two temperatures using the fractional order theory derived by Sherief *et al.*[9]. The uniqueness theorem, reciprocity theorem and a variational principle on this theory are also provided in the same article. Deswal and Kalkal[11] solved a two dimensional problem in a micropolar thermoviscoelastic medium by employing fractional order heat conduction with two temperatures.

The development of initial stress in the medium is due to many reasons such as the process of quenching, resulting from difference of temperatures, slow process of creep, differential external forces, and gravity variations. The earth is supposed to be under high initial stress. The researchers have shown much interest to study the effect of these stresses on the propagation of waves. Biot[12] solved the dynamic problem of elastic medium under initial stress. Chattopadhyay *et al.*[13] studied the reflection of elastic waves under initial stress at a free surface. Montanaro[14] studied the isotropic linear thermoelasticity with hydrostatic initial stress by using Biot's linearization of the constitutive law for stress. Othman and Song[15] investigated the reflection of plane waves from an elastic solid half-space under hydrostatic initial stress without energy dissipation. Recently, Yadav *et al.*[16] studied propagation of waves in an initially stressed generalized electromicrostretch thermoelastic medium with temperature-dependent properties under the effect of rotation.

The current manuscript is an attempt to discuss the phenomenon of wave propagation in a new theory of micropolar generalized thermoelasticity with fractional derivative heat transfer in an initially stressed thermodiffusive half space due to ramp type mechanical load, allowing the second sound effects. The medium is assumed initially quiescent. An analytical–numerical technique based on Laplace and Fourier transforms is adopted to solve the governing equations. Numerical results for temperature, concentration and stress distributions in physical space–time domain have been obtained for a magnesium crystal like material and presented graphically. Some comparisons have been shown in figures to estimate the effects of ramp parameter, fractional order parameter and initial stress on all the considered fields.

PROBLEM FORMULATION

Following Shaw and Mukhopadhyay[10], the field equations and stress-strain-temperature relations in an initially stressed rotating fractional order micropolar thermoelastic medium with diffusion are:

The constitutive relations:

$$\sigma_{ij} = \bar{\lambda} u_{r,r}\delta_{ij} + \bar{\mu}\left(u_{i,j} + u_{j,i}\right) + k\left(u_{j,i} - \varepsilon_{ijr}\phi_r\right) - \beta_1\theta\delta_{ij} - \beta_2 c\delta_{ij} - P(\delta_{ij} + \omega_{ij}) \,, \tag{1}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} \,, \tag{2}$$

Stress equation of motion:

$$\sigma_{ji,j} = \rho(\ddot{\vec{u}})_i, \tag{3}$$

Couple stress equation of motion:

$$\begin{aligned}&(\alpha+\beta+\gamma)\nabla\left(\nabla.\vec{\phi}\right) - \gamma\nabla\times\left(\nabla\times\vec{\phi}\right) + k\nabla\times\vec{u} - 2k\vec{\phi} - P\varepsilon_{imn}\omega_{mn} \\ &= \rho j\ddot{\vec{\phi}},\end{aligned} \tag{4}$$

Energy equation with fractional derivative heat transfer:

$$K^*\nabla^2\theta = \rho c_E\left(1+\tau_0\frac{\partial^m}{\partial t^m}\right)\frac{\partial\theta}{\partial t}+\beta_1 T_0\left(1+\tau_0\frac{\partial^m}{\partial t^m}\right)\frac{\partial}{\partial t}\nabla.\vec{u}+aT_0\left(1+\tau_0\frac{\partial^m}{\partial t^m}\right)\frac{\partial c}{\partial t}, \qquad (5)$$

Fractional order mass diffusion equation

$$Db\nabla^2 c = D\beta_2\nabla^2(\nabla.\vec{u})+Da\nabla^2\theta+\left(1+\tau_1\frac{\partial^m}{\partial t^m}\right)\frac{\partial c}{\partial t}, \qquad (6)$$

where

$$\omega_{ij}=\frac{1}{2}(u_{j,i}-u_{i,j}).$$

Here σ_{ij} denotes the components of force stress tensor, m_{ij} stands for the components of couple stress tensor, $\vec{\phi}$ and $\vec{u}$ are microrotation vector and displacement vector respectively and $\beta_1=\left(3\bar{\lambda}+2\bar{\mu}+k\right)\alpha_t$, $\beta_2=\left(3\bar{\lambda}+2\bar{\mu}+k\right)\alpha_c$ are the material constants, $\bar{\lambda}$ and $\bar{\mu}$ are generalized Lame's constants, satisfying the relations $\bar{\lambda}=\frac{E\upsilon}{\eta(1+\upsilon)(1-2\upsilon)}$ and $\frac{E}{\bar{\mu}}=2\eta(1+\upsilon)$. Here E,υ and η are Young's modulus, Poisson's ratio and initial stress parameter respectively. α,β,γ,k indicate micropolar material constants, j is the microinertia, α_t and α_c are coefficients of linear thermal expansion and linear diffusion expansion respectively, a,b,D are thermoelastic diffusion constants, τ_0,τ_1 are thermal and diffusion relaxation times respectively, $\theta=T-T_0$ and $c=C-C_0$, T is absolute temperature, T_o is temperature of medium in natural state, C is non equilibrium concentration, C_0 is mass concentration at natural state, ρ is the density of medium, K^* is the thermal conductivity, P is initial stress and m is the fractional order parameter such that $0<m\le 1$.

In the present context, let us consider an initially stressed micropolar thermodiffusive half space. We take *z* axis pointing vertically downward into the *xz* half space. We restrict our analysis to *xz* plane. For two-dimensional deformations, we have

$\vec{u}=(u,0,w),\ \vec{\phi}=(0,\phi_2,0).$

It is convenient to have the above equations rewritten in the dimensionless form. To this end, the following dimensionless parameters are introduced:

$$(x',z')=\frac{\omega^*}{c_1}(x,z),\ (u',w')=\frac{\rho\omega^* c_1}{\beta_1 T_0}(u,w),\ \theta'=\frac{\theta}{T_0},\ (t',\tau_0',\tau_1')=\omega^*(t,\tau_0,\tau_1),\ c'=\frac{c}{C_0},\ ,$$

$$\phi_2'=\frac{\rho c_1^2}{\beta_1 T_0}\phi_2,(\sigma_{ij}',P')=\frac{(\sigma_{ij},P)}{\beta_1 T_0},m_{ij}'=\frac{\omega^*}{c_1\beta_1 T_0}m_{ij},$$

where

$$\omega^*=\frac{\rho c_E c_1^2}{K^*},\quad c_1^2=\frac{(\lambda+2\mu+k)}{\rho}.$$

Using Helmholtz decomposition, the displacement components can be written as

$$u=\frac{\partial q}{\partial x}+\frac{\partial \psi}{\partial z},\quad w=\frac{\partial q}{\partial z}-\frac{\partial \psi}{\partial x}$$

where $q(x,z,t)$ and $\psi(x,z,t)$ are scalar potential functions.

Introducing the above dimensionless parameters and potential functions, equations (3) – (6) recast into the following forms (after dropping the prime):

$$\left(\nabla^2-\frac{\partial^2}{\partial t^2}\right)q-\theta-\varepsilon_1 c=0, \tag{7}$$

$$\left((a_0+a_1)\nabla^2-\frac{\partial^2}{\partial t^2}\right)\psi-a_0\phi_2=0, \tag{8}$$

$$\left(\nabla^2-2a_2-a_3\frac{\partial^2}{\partial t^2}\right)\phi_2+a_2\nabla^2\psi=0, \tag{9}$$

$$a_4\left[\frac{\partial}{\partial t}+\tau_0\left(\omega^*\right)^{m-1}\frac{\partial^{m+1}}{\partial t^{m+1}}\right]\nabla^2 q-\left[\nabla^2-\frac{\partial}{\partial t}-\tau_0\left(\omega^*\right)^{m-1}\frac{\partial^{m+1}}{\partial t^{m+1}}\right]\theta+a_5\left[\frac{\partial}{\partial t}+\tau_0\left(\omega^*\right)^{m-1}\frac{\partial^{m+1}}{\partial t^{m+1}}\right]c=0, \tag{10}$$

$$a_6\nabla^4 q + a_7\nabla^2\theta - \left[\nabla^2 - a_8\left(\frac{\partial}{\partial t} + \tau_1\left(\omega^*\right)^{m-1}\frac{\partial^{m+1}}{\partial t^{m+1}}\right)\right]c = 0, \tag{11}$$

where

$$\varepsilon_1 = \frac{\beta_2 c_0}{\beta_1 T_0},\ a_0 = \frac{k-S}{\rho c_1^2},\ a_1 = \frac{\mu_e}{\rho c_1^2},\ a_2 = \frac{kc_1^2}{\gamma\omega^{*2}},\ a_3 = \frac{\rho c_1^2 j}{\gamma},\ a_4 = \frac{\beta_1^2 T_0}{\rho K^*\omega^*},\ a_5 = \frac{aC_0c_1^2}{K^*\omega^*},$$

$$a_6 = \frac{\beta_1\beta_2 T_0}{\rho c_1^2 bC_0},\ a_7 = \frac{aT_0}{bC_0},\ a_8 = \frac{c_1^2}{Db\omega^*},\ S = \frac{\beta_1 T_0 P}{2}.$$

SOLUTION OF THE PROBLEM

Following the solution methodology through integral transform technique, we now operate Laplace and Fourier transforms on equations (7)-(11). The Laplace and Fourier transforms of a function $f(x,z,t)$ with parameters s and ξ are defined by the relations

$$\overline{f}(x,z,s) = \int_0^\infty f(x,z,t)e^{-st}dt, \tag{12}$$

$$\hat{f}(\xi,z,s) = \int_{-\infty}^{\infty}\overline{f}(x,z,s)e^{\iota\xi x}dx, \tag{13}$$

where over-bar and over-cap denote the Laplace and Fourier transforms respectively.

Applying the above transforms under homogeneous initial conditions on equations (7)-(11) and then solving, we obtain the following system of ordinary differential equations:

$$\left(D^6 + L_1D^4 + M_1D^2 + N_1\right)\{\hat{q},\hat{\theta},\hat{c}\} = 0, \tag{14}$$

$$\left(D^4 + L_2D^2 + M_2\right)\{\hat{\psi},\phi_2\} = 0, \tag{15}$$

where

L_1, M_1, N_1, L_2 and M_2 are defined in Appendix A.

The solutions of equations (14) and (15) can be expressed as

$$(\hat{q},\hat{\theta},\hat{c}) = \sum_{i=1,2,3}(1, a_i^*, b_i^*)R_i e^{-\lambda_i z}, \tag{16}$$

$$(\hat{\psi}, \hat{\phi}_2) = \sum_{i=4,5} (1, c_i^*) R_i e^{-\lambda_i z}, \tag{17}$$

where

$$\lambda_i^2 = 2\sqrt{-H}\cos\left(\frac{\theta + 2(i-1)\pi}{3}\right) - \frac{L_1}{3}, \ (i = 1, 2, 3)$$

$$\lambda_{4,5}^2 = \frac{-L_2 \pm \sqrt{L_2^2 - 4M_2}}{2},$$

$$H = \frac{1}{3}M_1 - \frac{1}{9}L_1^2, \ \ \theta = \tan^{-1}\left(\frac{\sqrt{|\Delta'|}}{-G}\right), \ \ \Delta' = G^2 + 4H^3, \ \ G = \frac{2}{27}L_1^3 - \frac{1}{3}M_1 L_1 + N_1,$$

$$a_i^* = \frac{g_4\lambda_i^2 - g_5}{g_2\lambda_i^2 + g_3}, \ \ b_i^* = \frac{\lambda_i^4 - g_6\lambda_i^2 + g_7}{g_2\lambda_i^2 + g_3}, \ \ c_i^* = \frac{[(a_0 + a_1)(\lambda_i^2 - \xi_1^2) - s^2]}{a_0}$$

and $R_{(i=1,2,3,4,5)}$ are unknown constants depending upon s and ξ.

BOUNDARY CONDITIONS

The surface of the half space is subjected to ramp type mechanical load. The corresponding boundary conditions can be described as

$$\sigma_{zz} + P = -\sigma_1 \delta(x) h(t), \sigma_{zx} = m_{zy} = \theta = c = 0 \text{ at } z = 0, \tag{18}$$

where σ_1 represents the strength of load and $\delta(x)$ is the Dirac-delta function and $h(t)$ is defined as

$$h(t) = \begin{cases} 0, & t \le 0 \\ \dfrac{t}{t_0}, & 0 < t \le t_0, \\ 1, & t > t_0 \end{cases}$$

t_0 is ramp parameter.

Making use of these boundary conditions, we obtain the following values for the displacement components, stresses, temperature field and mass concentration

$$\hat{u} = -\frac{1}{\Delta}\left[\iota\xi\sum_{i=1,2,3}\Delta_i e^{-\lambda_i z} + \sum_{i=4,5}\lambda_i\Delta_i e^{-\lambda_i z}\right], \tag{19}$$

$$\hat{w} = -\frac{1}{\Delta}\left[\sum_{i=1,2,3}\lambda_i\Delta_i e^{-\lambda_i z} - \iota\xi\sum_{i=4,5}\Delta_i e^{-\lambda_i z}\right], \tag{20}$$

$$\hat{t}_{zz} = \frac{1}{\Delta}\left[\sum_{i=1,2,3} r_i\Delta_i e^{-\lambda_i z} - \iota\xi(1-b_2)\sum_{i=4,5}\Delta_i e^{-\lambda_i z}\right] - P, \tag{21}$$

$$\hat{t}_{zx} = \frac{1}{\Delta}\left[\iota\xi_1\sum_{i=1,2,3}\lambda_i\Delta_i e^{-\lambda_i z} + \sum_{i=4,5} r_i\Delta_i e^{-\lambda_i z}\right], \tag{22}$$

$$\hat{m}_{zy} = -\frac{b_4}{\Delta}\left[\sum_{i=4,5} c_i^*\lambda_i\Delta_i e^{-\lambda_i z}\right], \tag{23}$$

$$\hat{\theta} = \frac{1}{\Delta}\left[\sum_{i=1,2,3} a_i^*\lambda_i\Delta_i e^{-\lambda_i z}\right], \tag{24}$$

$$\hat{c} = \frac{1}{\Delta}\left[\sum_{i=1,2,3} b_i^*\lambda_i\Delta_i e^{-\lambda_i z}\right], \tag{25}$$

where all the constants are defined in Appendix B.

SPECIAL CASE

Neglecting fractional effect

To discuss the problem of wave propagation in a generalized thermoelasticity theory of integer type with initial stress and diffusion, it is sufficient to ignore the fractional values of m which is the fractional order parameter. For this purpose we will take $m=1$ in basic equations. Hence, the problem reduces to the well-known conventional problem of generalized thermoelasticity and values of all the fields can be procured from the expressions (19) -(25) by making suitable modifications in the governing equations.

INVERSION OF THE TRANSFORMS

The transformed components of displacements, temperatures and stresses can be formally expressed as function of z and the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $\hat{f}(\xi, z, s)$. In order to obtain the solution of the problem in the physical domain, we invert the double transforms in equations (19)-(25) by adopting the methodology of Rakshit and Mukhopadhyay [17].

NUMERICAL RESULTS AND DISCUSSION

With an aim to illustrate the contribution of different parameters on the field quantities, a numerical analysis is carried out. To fulfill this purpose, we have chosen magnesium crystal like parameter, for which

$$\rho = 1.74\times10^{3} kgm^{-3},\ E = 10.8\times10^{10} kgm^{-1}s^{-2},\ \upsilon = 0.35,$$

$$k = 1.7\times10^{2} Jm^{-1}s^{-2},\ \gamma = 0.779\times10^{-9} kgms^{-2},\ j = 0.2\times10^{-19} m^{2},$$

$$K^{*} = 1.0\times10^{10} Nm^{-2},\ c_{E} = 1.04\times10^{3} Jkg^{-1}K^{-1},\ T_{0} = 298K,\ \tau_{0} = 0.1s,$$

$$a = 1.2\times10^{4} m^{2}s^{2}K^{-1},\ b = 0.9\times10^{6} m^{5}kg^{-1}s^{-2},\ \tau_{1} = 0.2s,$$

$$D = 0.85\times10^{-8} kgsm^{-3},\ P = 1,\ \eta = 2.5,\ \sigma_{1} = 5,\ t_{0} = 0.5,\ \alpha_{t} = 2.36\times10^{-5} K^{-1},\ \alpha_{c} = 1.98\times10^{-4} m^{3}kg^{-1}.$$

With these numerical values of the parameters, the values of the field quantities are computed for the time t = 0.1. From application point of view, we have divided the graphs into tho groups. The first group (Figures 1-4) exhibits effect of initial stress and fractional parameter on different field variables. When initial stress is neglected, we have taken $P = 0,\ \eta = 1$ in field equations while $m = 1$ correspond to without fractional effect case. In the second group (Figures 5-8) field variables are plotted for three different values of ramp parameter (0.3, 0.5 and 0.9).

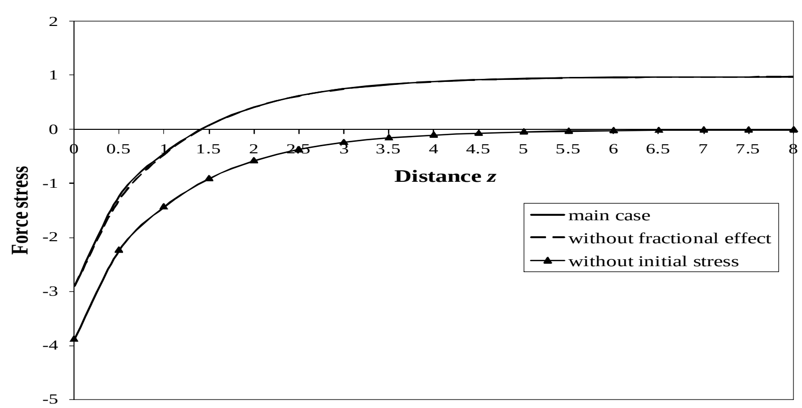

Fig. 1. Effect of fractional parameter and initial stress on force stress

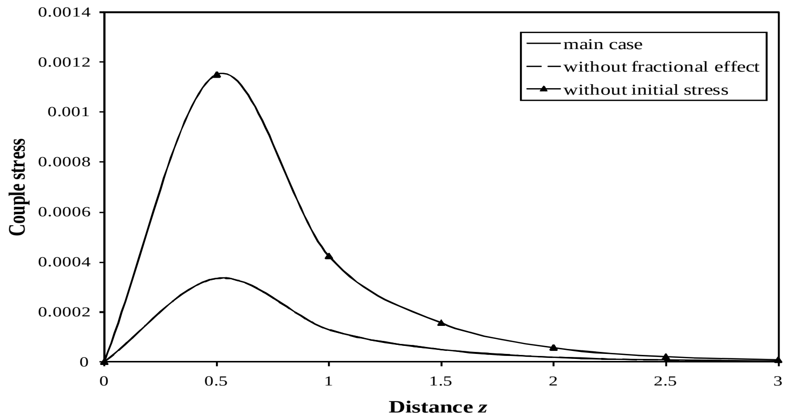

Fig. 2. Effect of fractional parameter and initial stress on couple stress

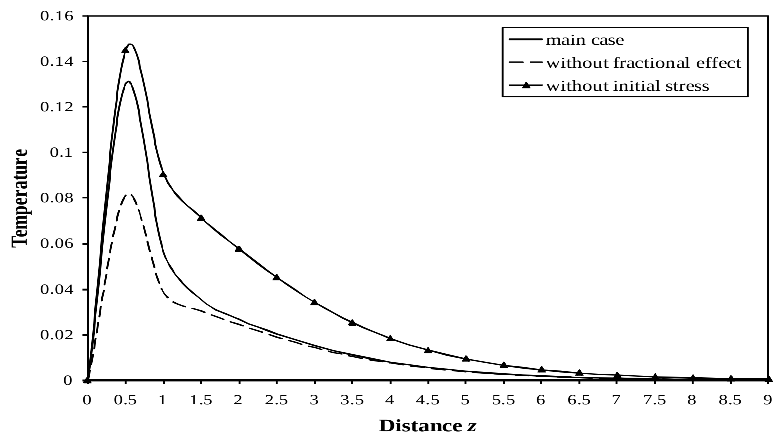

Fig. 3. Effect of fractional parameter and initial stress on temperature

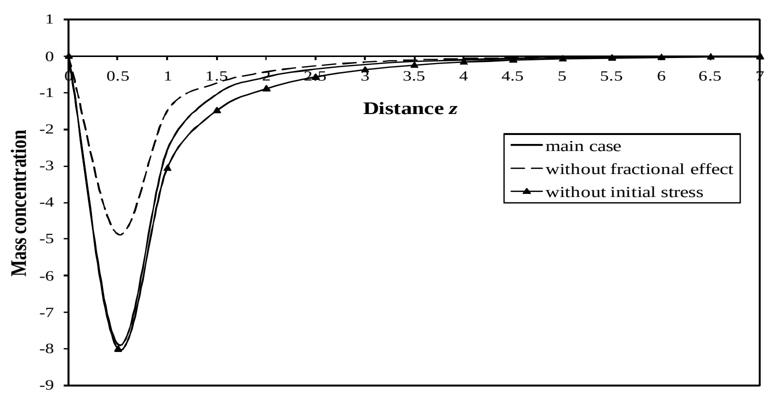

Fig. 4. Effect of fractional parameter and initial stress on mass concentration

Group I

Figure 1 shows effects of fractional parameter and initial stress on force stress. It is observed that the most negative value of couple stress is gained at *z*=0. Presence of initial stress and fractional parameter exhibit mix kind of effect on numerical values of force stress. One more interesting observation is that in the absence of initial stress, force stress ultimately tends to zero while it tends to non zero constant value in the presence of initial stress.

Figure 2 depicts profile of couple stress against distance z in order to check the effects of fractional parameter snd initial stress. The profile of couple stress is similar in all the three cases. All the three curves start with zero satisfying boundary conditions. Then attain its maximaum value near *z*=0.5 and then ultimately tend to zero. Initial stress has decreasing effect while fractional parameter has increasing effect on couple stress although the curves in the presence and absence of fractional parameter seem to be coinciding.

Figure 3 and Figure 4 describe effects of fractional parameter and initial stress on temperature and mass concentration respectively. Initially, the values of both these fields start with zero which is in accordance with the boundary conditions. Fractional parameter acts as an increasing agent while initial stress acts as a decreasing agent for temperature field which is clear from Figure 3. The same scenario is observed for mass concentration in Figure 4. Also it is observed that both these fields attain their maximum numerical value near *z*=0.5 and then ultimately tend to zero in all the cases.

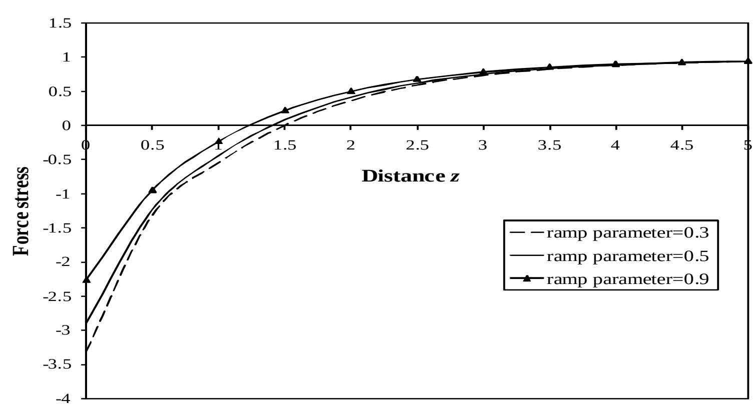

Fig. 5. Effect of ramp parameter on force stress

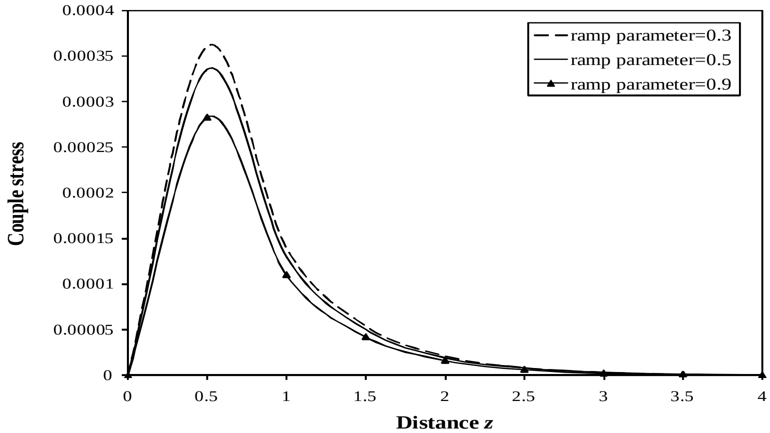

Fig. 6. Effect of ramp parameter on couple stress

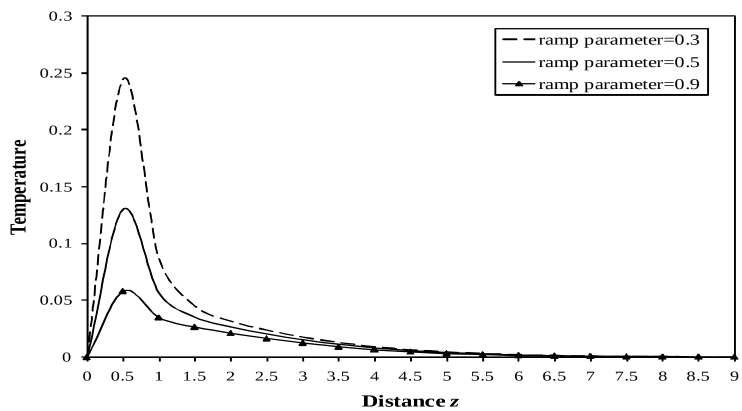

Fig. 7. Effect of ramp parameter on temperature

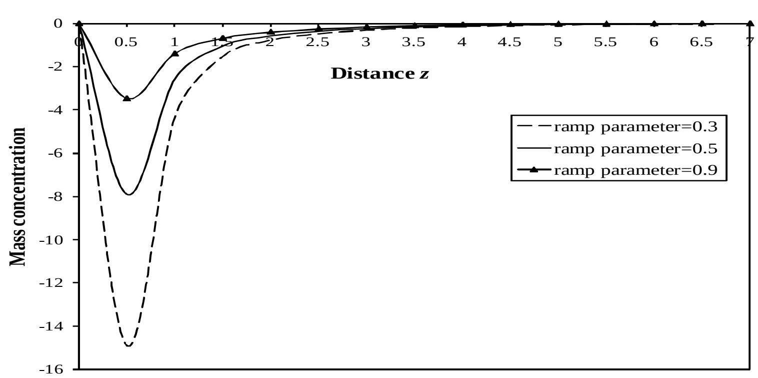

Fig. 8. Effect of ramp parameter on mass concentration

Group II

Figure 5 and Figure 6 describe effect of ramp parameter on force stress and couple stress respectively by taking three values of ramp parameter (0.3, 0.5 and 0.9). It is clear from Figure 5 that ramp parameter shows miscellaneous effects on numerical values of force stress. Figure 6 shows that the profile of couple stress is similar for all the three values of ramp parameter. It is observed that ramp parameter has decreasing effect on couple stress.

Figure 7 and Figure 8 are plotted to evince the effect of ramp parameter on temperature field and mass concentration field respectively. For all the three values of ramp parameter, both these fields vanish at z=0 satisfying the boundary conditions. Increment in the values of ramp parameter minifies the values of temperature and mass concentration numerically. Thus ramp parameter has decreasing effect on both these fields.

CONCLUDING REMARKS

The main goal of this work is to introduce a new mathematical model of heat conduction with time fractional order m for isotropic material as an improvement and progress in the field of micropolar thermoelasticity. The reason of this development is that a fractional model can describe simply and elegantly the complex characteristics of a thermoelastic material. According to the above analysis, we can conclude the following points:

- The phenomenon of finite speed of propagation is manifested in all the figures except the force stress.
- It is interesting to notice from Figure 1 that in the absence of initial stress, force stress also tend to zero.
- Initial stress acts as a decreasing agent for all the physical fields except force stress.
- The effect of ramp parameter on all the studied fields is very much significant.
- It is apparent from figures that presence of fractional parameter magnifies all the field variables numerically except force stress.

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APPENDIX A

$$L_1=\frac{a_6(g_3-2g_2\xi^2)+a_7g_4+\xi^2+a_8g_1'+g_6}{a_6g_2-1},\ M_1=\frac{a_6(g_2\xi^2-2g_3)\xi^2-a_7(g_4\xi^2+g_5)-g_6(\xi^2+a_8g_1')-g_7}{a_6g_2-1},$$

$$N_1=\frac{a_6g_3\xi^4+a_7g_5\xi^2+g_7(\xi^2+a_8g_1')}{a_6g_2-1},$$

$$L_2=-\left[2\xi^2+2a_2+a_3s^2+\frac{s^2-a_0a_2}{a_0+a_1}\right],M_2=(\xi^2+2a_2+a_3s^2)\xi^2+\frac{s^2(2a_2+a_3s^2)+\xi^2(s^2-a_0a_2)}{a_0+a_1},$$

Where

$$g_1=s\left(1+\frac{\tau_0(\omega^*s)^m}{\omega^*}\right),\ g_1'=s\left(1+\frac{\tau_1(\omega^*s)^m}{\omega^*}\right),\ g_2=\varepsilon_1,\ g_3=a_5g_1-g_1(\xi^2+g_1),$$

$$g_4=g_1(a_5+g_2a_4),\ g_5=g_4\xi^2+a_5g_1s^2,\ g_6=2\xi^2+g_1+s^2+a_4g_1,\ g_7=(\xi^2+g_1)(\xi^2+s^2)+a_4g_1\xi^2.$$

APPENDIX B

$$\Delta=(1-b_2)(c_4^*-c_5^*)\xi\xi_1\lambda_4\lambda_5(\lambda_1f_1-\lambda_2f_2+\lambda_3f_3)+(c_5^*\lambda_5r_4-c_4^*\lambda_4r_5)(r_1f_1-r_2f_2+r_3f_3),$$

$$\Delta_1=-\sigma_1f_1\bar{F}(s)(c_5^*\lambda_5r_4-c_4^*\lambda_4r_5),\ \Delta_2=\sigma_1f_2\bar{F}(s)(c_5^*\lambda_5r_4-c_4^*\lambda_4r_5),\ \Delta_3=-\sigma_1f_3\bar{F}(s)(c_5^*\lambda_5r_4-c_4^*\lambda_4r_5),$$

$$\Delta_4=\iota\sigma_1\bar{F}(s)\xi_1c_5^*\lambda_5(\lambda_1f_1-\lambda_2f_2+\lambda_3f_3),\ \Delta_5=-\iota\sigma_1\bar{F}(s)\xi_1c_4^*\lambda_4(\lambda_1f_1-\lambda_2f_2+\lambda_3f_3),$$

$$r_i=\lambda_i^2-b_2\xi^2-a_i^*-g_2b_i^*,\qquad (i=1,2,3)$$

$$r_{4,5}=(a_0+b_3)\lambda_{4,5}^2+(b_3+k_0)\xi^2-d_0'\,c_{4,5}^*,$$

$$f_1 = a_2^* b_3^* - a_3^* b_2^*,\ f_2 = a_1^* b_3^* - a_3^* b_1^*,\ f_3 = a_1^* b_2^* - a_2^* b_1^*,$$

$$\xi_1 = \xi(2b_3 + a_0 + k_0),\ k_0 = \frac{\beta_1 T_0}{2c_1^2},\ a_0' = \frac{k}{\rho c_1^2},\ b_2 = \frac{\lambda}{\rho c_1^2},\ b_3 = a_1,\ b_4 = \frac{\gamma \omega^{*^2}}{\rho c_1^4}$$

and $\overline{F}(s) = \dfrac{(1 - e^{-st_0})}{t_0 s^2}.$