A Novel Approach for the calculation of Townsend Coefficients in Argon Glow Discharge Plasmas

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ABSTRACT

Breakdown voltage of a gas is the required voltage to start a discharge or electric arc through the gas. Paschen’s law describes the characteristics of gas breakdown voltage between two electrodes. This law states that the Gas breakdown voltage \( V_B \) depends only on the product of gas pressure \( p \) and gap length \( d \) between electrodes \( (V_B=f(pd)) \). This paper discuss about the variation of the two important coefficients involved in the Paschen’s law, the first Townsend coefficient \( (\alpha) \) and second Townsend Coefficient \( (\gamma) \) with the fundamental parameter, reduced field \( E/p \). A gas discharge system with a large gap length compared to electrode radius is used for the study. The role of inter-electrode distance \( (d) \), electrode radii \( (r) \) and the \( d/r \) ratio on these Coefficients have successfully established. The results reported graphically and detailed explanations for the findings were included.

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1. INTRODUCTION

When a sufficiently high potential difference is applied between two electrodes placed in a gas, the latter will break down into positive ions and electrons, giving rise to a gas discharge\(^1\). The electrons are accelerated by the electric field in front of the cathode and collide with the gas atoms. The ionization collisions create new electrons and ions. The ions are accelerated by the electric field toward the cathode, where they release new electrons by ioninduced secondary electron emission. The electrons give rise to new ionization collisions, creating new ions and electrons. These processes of electron emission at the cathode and ionization in the plasma make the glow discharge a self-sustaining plasma\(^1,2\).

The breakdown mechanism in gases can be explained using Paschen law. Paschen law connects the breakdown voltage for a particular gas between two electrodes of distance (d) in a discharge tube with the gas pressure(p) as follows \(^3,4\).

\[
V_B = \frac{Bpd}{\log_e(Apd) - \log_e(Ae^{(1+1/p)})} \quad \text{(1)}
\]

Where A is called the saturation ionization in the gas and B is a constant related with the excitation and ionization energies. Both A and B are gas dependent constants defined for a particular \(E/p\) ratio, where E is the electric field. The \(E/p\) signifies the electric stress per pressure \(^5,6\).

The secondary emission yield or second Townsend coefficient \(\gamma\) represents the number of electrons emitted per incident positive ion from cathode, which depends on the material which is used as cathode, (since each material has different work function), and also on ion energy and state of the surface. The Townsend condition for breakdown can be written as;

\[
\gamma(e^{ad} - 1) = 1 \quad \text{(2)}
\]

Where \(\alpha\) is called the first Townsend coefficient or rate of ionization per unit length \(^3,4,6\). The dependence of electric field (E) on \(\alpha\) can be written as

\[
\alpha = Ape^{(-Bp/E)} \quad \text{(3)}
\]

This expression is an empirical formulae used in theoretical and numerical analysis of gas discharges obtained by John Sealy Edward Townsend \(^6\). Using these expressions we can obtain the value of \(\alpha\) and \(\gamma\) for the known \(E/p\) range of A and B.

In this work, we used argon as the working gas and stainless steel as electrode material. The first and second Townsend coefficients are evaluated for a reduced field range of 100 – 200 V/Torr-cm. To the best of my knowledge and belief, the dependence of first Townsend coefficient \(\alpha\) and second Townsend coefficient \(\gamma\) on electrode distance and radius for large discharge gaps has not studied yet. The importance of obtaining the variation of \(\gamma\) with the reduced electric field \((E/p)\) has
practical applications. Plasma display panels used in flat screen televisions are cells containing plasma. The variation of $\gamma$ with $E/p$ should be understood to gain the proper idea on the physical process that involved inside these plasma cells.$^{3,14}$

2. FIRST TOWNSEND COEFFICIENT (\(A\))

The mean free path for ionization is defined as the distance travelled by an electron between ionizing collisions. Its inverse represents the number of ionizing collisions that takes place in a centimetre, which is called the first Townsend coefficient, and its unit is cm\(^{-1}\). Townsend introduced the coefficient $\alpha$ to explain the exponential rise in current. The gas gain can be obtained using this fundamental parameter.$^{4,7}$ This fundamental parameter can be used to describe the growth of charge in a detector which operates in avalanche mode. For validating electron collision cross section and for modelling discharge the parameter $\alpha$ plays an important role, which is beyond its practical application. The conditions of discharge ignition can be obtained from the magnitude of first ionization coefficient, so the evaluation of this coefficient is very important. The operating point in a gas discharge is established at a point where losses are compensated by ionization and for that $\alpha$ is a critical parameter which is used to model plasma maintenance.$^{8,9,11}$

The nature of gas, electric field strength and gas pressure are the main factors on which first Townsend coefficient depends. The fundamental parameter of gas discharge is the reduced field $E/p$ ratio. The parameters that involved in gas discharge (almost all) depends on reduced field $E/p$. With the increase in electric field, $\alpha$ increases smoothly and also it depends on the pressure. The empirical relation of $\alpha$ for gas discharges by Townsend can be written as: 
$$E/p = A e^{(-Bp/E)}.$$ From the above equation it is clear that $\alpha/p$ is a function of $E/p$, for a wide range of $E$ and $p$. $^{7,8,9}$ The first Townsend coefficient related with the reduced field by the similarity law as;
$$\frac{\alpha}{p} = f \left( \frac{E}{p} \right) \quad (4)$$

The accurate value of Townsend coefficient is reported in a limited range of parameters for some common gases.$^{11,12}$

The value of $A$ and $B$ have reported by many authors but for a range of 100-600 V/Torr-cm.$^6$ Using these values we can obtain the plot between $\alpha$ and E/p. In some another literature, it has been given that in the case of Argon for an E/p less than 11.8 V/Torr-cm the value of $A$=2.8 and $B$=60.6 [7]. So we can obtain data points for E/p below 11.8 (Torr-cm). The Value of $A$ and $B$ for a range of E/p between 11.8 V/(Torr- cm) and 100 V/torr-cm is unknown. So an interpolation technique is used to obtain values of $\alpha$ between the range of 11.8 and 100 V/(Torr-cm).
3. SECOND TOWNSEND COEFFICIENT ($\gamma$)

The sustainability of gas discharge is due to the emission of secondary electrons from cathode. This secondary emission process can be explained as the number of secondary electrons emitted from the cathode per incident particle on the cathode, and is called the secondary emission yield or second Townsend coefficient. This secondary electron emission mechanism is the most important mechanism in discharge operation. The secondary electrons are produced by the collision of different species such as ions, meta-stable atoms, photons, fast moving atoms etc with the cathode material. Therefore we can say that $\gamma$ has great dependence on the type of material that is used as electrode, more precisely the work function of the material. For a material with less work function the value of $\gamma$ will be high, under identical experimental condition. The type of gas used and the reduced field $E/p$ are also factors on which $\gamma$ depends on $^4,10,14$.

All the factors ion collision, photon collision, meta-stable atom collision with the cathode contributes the total secondary electron yield, and their individual contribution collectively adds together to obtain the total yield produced. The dominant process for secondary electron emission varies with the $E/p$ ratio. The dominance of the contribution of production of secondary electron by ions is limited only in a short reduced electric field range. The interest of studying the secondary electron coefficient, leads to identify the contribution of all process involved in secondary electron emission process and the ions causing secondary electron emission (assumption according to the theory put by Townsend) is limited to very small range conditions, and for much wider range of electric field gas phase ionization caused by neutrals and photons can be identified. Many authors have reported the secondary electron emission coefficient as a function of $E/p^{4,9,17}$.

4. RESULT AND DISCUSSION

From the Townsend condition for breakdown as given in equation 2, we can calculate the $\gamma$ values, for that we need the values of $\alpha$. The interpolated graphs give the full range values of $\alpha$. The first Townsend coefficient $\alpha$ values for the full range were calculated by interpolation for different electrode distance, different electrode radius and also by keeping $d/r$ as constant. Using these values the second Townsend coefficient were studied for these three different conditions.

4.1 Variation of First Townsend Coefficient ($\alpha$) with $E/p$ for Different Electrode Distances

The variation of first Townsend coefficient ($\alpha$) with $E/p$ for different electrode distances and constant electrode radii are plotted in figure (1). As distance increases, the first Townsend coefficient ($\alpha$) decreases for the same $E/p$ value.
Figure 1: Variation of first Townsend coefficient ($\alpha$) with ($E/p$) for variable inter-electrode distances.
Figure 2: Interpolated graphs for first Townsend coefficient ($\alpha$) with E/p to obtain the variation of plots in the whole range

The value of constants A and B are available for low E/p range also. Using this value, we calculated the first Townsend coefficient ($\alpha$) and then interpolated for the middle range values. The interpolated graphs are shown in figure 2. The variation of $\alpha$ with E/p for different electrode separation distances for full range(0-300 V/(Torr$\cdot$cm) is shown in figure 3. The variation of first Townsend coefficient ($\alpha$) with reduced electric field shows a strong dependence on distance. For low reduced field value, the ionization is little and excitation is high because of low value of electric field and high value of pressure. Low electric field cannot ionize all the gas particles indicating lesser value of ionization coefficient. As reduced field increases, either electric field increases or pressure decreases, so that ionization rate is enhanced. First Townsend coefficient ($\alpha$) is defined as the number of ion pair produced on average by an electron traversing a distance 1cm.

Figure 3: Variation of Townsend coefficient ($\alpha$) with (E/p) for variable distance with whole range of E/p
As distance increases, electrons have to travel more to produce the same number of ion pairs for the same reduced field. Also by means of diffusion transverse to the electric field towards the discharge tube side walls, the charged particles can be lost from the system on travelling more through the system, causes a reduction of ionization rate. Due to all these effects, as distance increases, the first Townsend coefficient (α) decreases for the same E/p value.

4.2 Variation of First Townsend Coefficient (α) with E/p for different electrode radius

The variation of first Townsend coefficient (α) with E/p for different electrode radii at constant distances is plotted in figure 4 for a range of 100-200 V/Torr-cm. The curves found to be superimposed at the low E/p range, but deviates in the high E/p range.

![Figure 4: Variation of Townsend coefficient (α) with (E/p) for variable electrode radius](image)

The value of constants A and B are available for low E/p range also. Using this value, we calculated the α and then interpolated for the middle range values. The interpolated graphs are shown in figure 5. The variation of α with E/p for different electrode radius for full range(0-300 V/(Torr-cm) is shown in figure 6.
The shape of the curves obtained in figure 6 can explain as same in section 3.4. For larger electrode radius there should be more seed electrons present and so the ionization collisions also high. Therefore mean free path is less and so its inverse first Townsend coefficient ($\alpha$) have relatively high value as the electrode radius increases. But for lower reduced field due to the low value of electric field, the ionization is less and so the effect of seed electrons is not significant.
4.3 Variation of first Townsend coefficient (α) with E/p for same d/r ratio

The variation of first Townsend coefficient (α) with reduced electric field (E/p) for same d/r ratio is plotted in figure 7. The obtained plot shows significant variation for the curves with electrode separation distances and electrode radius, even though their ratio is kept constant.

Figure 7: Variation of first Townsend coefficient (α) with E/p for constant d/r ratio

The value of constants A and B are available for low E/p range also. Using this value, we calculated the alpha and then interpolated for the middle range values. The interpolated graphs for all the three cases are separately shown in figure 8. Using the interpolation data the variation of alpha with reduced field for different electrode separation distances for full range is shown in figure 9.

Figure 8: Interpolated graphs for all d/r ratio

Figure 9: Variation of first Townsend coefficient (α) with E/p for full range
In the study of first Townsend coefficient for different electrode separation keeping electrode radius constant, the value of First Townsend Coefficient ($\alpha$) is relatively low for higher electrode distances because of diffusion losses while travelling longer distances. Here also the curves in figure 9 show the same effects.
Also, in the study of first Townsend coefficient for different electrode radii keeping electrode separation distance constant, relatively larger value of $\alpha$ are for electrode with larger radius, due to more seed electrons. While we examining curves in figure 9 it is evident that this effect of more number of seed electrons in higher electrode radii are not leads to relatively high value of $\alpha$. The reason is that, for keeping $d/r$ ratio as constant, as the radii increases, the electrode separation distance also increases and so the diffusion losses are high. From this study, it is clear that the dependence of alpha on electrode radius is very little, or negligible, as compared with its dependence on electrode separation distances. The figure 10 shows the variation of first Townsend coefficient with reduced field is a combined figure of all cases under my study and it strongly support the above arguments.
4.4 Variation of second Townsend Coefficient ($\gamma$) with $\frac{E}{p}$ for Different Electrode Distances

The variation of Second Townsend Coefficient($\gamma$) with reduced field($\frac{E}{p}$), for stainless steel electrode and argon gas, for a separation distance less than 6cm are plotted by Noori and Ranjbar\textsuperscript{20}. The value of $\gamma$ at 100 V/Torr-cm for an inter-electrode distance of 6cm is slightly above $10^{-3}$ and the value is rising with decrease in gap length\textsuperscript{20}. The $\gamma$ value obtained in this case is less than this, due to the fact that the gap length under our observation is large.

![Figure 10: Variation of Townsend coefficients ($\alpha$) with ($E/p$) for constant d/r ratio for whole range of $E/p$ using interpolation](image1)

![Figure 11: Variation of second Townsend coefficient ($\gamma$) as a function of $\frac{E}{p}$ for different distances](image2)
In the left side of the curve second Townsend coefficient ($\gamma$) rapidly decreases with increasing E/p ratio. The main contribution of secondary electron at this region is the photons that interacting with the cathode. Excited atoms, due to the inelastic collision of neutral atoms with electron are responsible for the production of photons (at least one photon is emitted by each excited state). As E/p ratio is low, the field is low and collision causes more excitation than ionization, so that the value of secondary electron is high. But on increasing the E/p ratio, the photo electric emission decreases, and the $\gamma$ value also decreases. The positive ion bombardment with cathode may be probable process in the high E/p region (but very feeble). At low E/p value, the meta-stable excited atoms also contributes for secondary emission along with photoelectric emission $^{14,17,20}$.

The second Townsend coefficient ($\gamma$) suddenly decreases for small increase of E/p value in the lower value of reduced field. In the figure 11 $\gamma$ varies from 7.33 to $5.8 \times 10^{-7}$. This is a large variation and so the gamma value for E/p above around 50 V/(Torr-cm) seems to be constant. But plotting separately the gamma vs E/p for E/p less than 100 V/(Torr-cm) and greater than 100 V/(Torr-cm), as shown in figure 12 and 13 shows that the gamma value is not constant. The curves show a trend that for higher reduced field, $\gamma$ approaches to zero.

![Figure 12: Variation of second Townsend coefficient ($\gamma$) as a function of E/p over a range of 0-100 V/(Torr-cm)](image)
As distance increases, the value of $\gamma$ is found to be decreasing. Malik N. H (1978) stated the strong decrease in second Townsend coefficient value with increase in distance. The distance increases, the rate of secondary ionization is decreased. This can be attributed to the fact that the decrease in photoelectric effect due to photon collision with the cathode, as it has to travel a large distance to reach the cathode, the photon absorption in the gas medium is enhanced by increasing the gap length, which reduces the value of second Townsend coefficient $^{14,21}$.

4.5 Variation of Second Townsend Coefficient ($\gamma$) with $E/p$ for different electrode radius

The variation in second Townsend coefficient ($\gamma$) with $E/p$ for different electrode radius is plotted in figure 6. The variation of second Townsend coefficient ($\gamma$) with $E/p$ for the whole range of $E/p$ for different radius does not reveals anything as the value of $\gamma$ varies over a wide range. So we have to split the range of $E/p$ into two, from 0 to 100 V/(Torr-cm) and 100-250 V/(Torr-cm) as shown in figure 7 and 8.
Figure 14: Variation of Second Townsend Coefficient ($\gamma$) as a function of $E/p$ for different radius

Figure 15: Variation of Second Townsend Coefficient ($\gamma$) as a function of $E/p$ over a range of 0-100 V/(Torr-cm)
The curves are almost overlapped in the very left side. The contribution of secondary electron emission from cathode is not only due to photons but meta-stable particles, the contribution of photon is almost ceased when comparing with the low E/p region (figure 15). Since secondary electron generators are very less, the radius has no characteristics influence. The distance travelled by the photon to reach the cathode is same in all the three cases are same and therefore equal generation of secondary electrons and hence the curves overlapped. On close observation for E/p greater than 120 V/(Torr-cm), one with the least radius value has the relatively greater γ value, and that with the large radius electrode has the relatively smaller gamma for same E/p value.

As in figure 6 the variation of first Townsend coefficient(α) with E/p for constant distance and variable radius was analyzed. On that case for r= 5cm, the first Townsend coefficient (α) is relatively high when comparing with r=4cm, and for r=2.5cm, the value of first Townsend coefficient is the lowest. Here for second Townsend coefficient (γ), this observation is reversed. That is for r=2.5 cm, second Townsend coefficient is high comparing with r=4cm and the lowest for r=5cm, this can be explained on the self sustaining condition proposed by Townsend $\gamma = \frac{1}{(\alpha d - 1)}$, the equation suggest that for self sustaining discharge there should be a charge balance between the number of primary electrons and secondary electrons. The secondary electrons generated are in such a way that both the number of secondary electrons and primary electrons (that remains after moving towards anode) should be the same in all the three cases(r=5cm, r=4cm and r=2.5cm). For r=5cm, the primary electron coefficient is high comparing with r=4cm, so the secondary electron emission is small comparing with r=4cm. For r=2.5 cm, primary electron generated is the lowest when
comparing with \( r=4 \) cm and \( r=5 \) cm, therefore the value of secondary electron coefficient is the highest for \( r=2.5 \) cm.

4.6 Variation of Second Townsend Coefficient (\( \gamma \)) with \( E/p \) for same \( d/r \) ratio

The variation in second Townsend coefficient (\( \gamma \)) with reduced electric field\( (E/p) \) for constant \( d/r \) ratio is obtained in figure 17 for the whole range of \( E/p \).

![Figure 17: Variation of second Townsend coefficient (\( \gamma \)) as a function of reduced field \( E/p \) with constant \( d/r \)](image)

As the value of second Townsend coefficient is varying in a wide range, this single plot cannot be used for the analysis. So we plot the variation in two figures. In figure 18, for an \( E/p \) range of 0-100 \( V/(Torr\cdot cm) \) and in figure 19, for an \( E/p \) range of 100-250 \( V/(Torr\cdot cm) \). In the low \( E/p \) range, the curves are not found to be overlapped, but for higher \( E/p \) value the curves are found to be overlapping.

![Figure 18: Variation of second Townsend coefficient (\( \gamma \)) as a function of reduced field \( E/p \) over a range of 0-100 \( V/(Torr\cdot cm) \)](image)
In the low E/p range, the secondary electron emission, in this region is mainly due to photoelectric emission, figure 18. The probability of a photon reaching the cathode is less in the case of longer distance, when comparing with a shorter gap. But more photon can interact with larger radii, to generate secondary electrons. For higher radius, even if the gap length is large generates more secondary electrons. From figure 19 the left side of the curves clearly shows no effect for the d/r ratio, the effect of radius is pronounced in that region, the photons can interact more on cathode with higher radius. As the value of E/p increases the photoelectric effect becomes completely ceased and the ion induced secondary emission can probably occur.

The charged particle on travelling more distance can be lost by means of diffusion, but the radius is larger in that case, so that more particles can interact with cathode to generate secondary electrons. For smaller distance case, diffusion losses are less, and interaction area on cathode is also less, keeping a constant secondary electron generation in the high E/p range.

5. CONCLUSIONS

The First Townsend coefficient(α) and second Townsend coefficient(γ) are very important factors in plasma maintenance since plasma is maintained in gas discharge by means of ionization rate. The electric field (E) or pressure (p) is not the fundamental parameter of gas discharge instead reduced electric field (E/p) is fundamental. The variation of first Townsend coefficient (α) with E/p is analyzed by changing the distance, radius and for constant d/r ratio. The graphs are plotted and the results are analyzed. The results indicate a characteristic dependence of electrode distance on first Townsend coefficient (α), rather than the electrode radius.
The variation of $\gamma$ with $E/p$ is studied. Almost all parameters in gas discharge depend on the reduced field. The influence of electrode distance and radius are analyzed and plotted $\gamma$ versus $E/p$ curves. The result indicates the characteristic dependence of the curves on electrode separation distance, and negligible dependence on electrode radii.

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