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[Interval Valued Vague ILI – Ideals of Lattice Implication Algebras

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ABSTRACT

The concept of interval valued vague ILI – ideals of lattice implication algebras was introduced. The relationship among interval valued vague ILI – ideals, interval valued vague LI – ideals and interval valued vague lattice ideals was studied. The relation between interval valued vague ILI – ideals and its cut sets was discussed. Extension property of interval valued vague LI – ideals ILI – ideal is built.

KEYWORDS- Lattice Implication Algebras, Interval valued vague ILI – ideals, Interval valued vague LI – ideals and Interval valued vague lattice ideals.

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I. INTRODUCTION

In order to research the logical system whose proportional value is given lattice, Y. XU[7] proposed the concept of lattice implication algebras, and discussed their some properties. Y.XU, Y.B. Jun and E.H. Roh introduced the notion of LI – ideals of a lattice implication algebras, and discussed their some properties.In particular Young Lin Liu, Yang Xu, Qin and Liu[6] introduced the notion of ILI – ideals of lattice implication algebras.

Vague set theory was first introduced by Gau and Buehrer[4] in1993. The vague set is an extension of fuzzy set. A vague set H in the universal of discourse U is characterized by a truth membership function t_A and a false membership function f_A. Actually, vague sets can realistically reflect the actual problem. But more often, the truth-membership and false-membership are in a range. For this reason, the notion of interval valued vague sets was presented by Atanassov in 1989[1]. And it is regarded as an extension of the theory of vague sets. In this theory, the truth-membership function and false-membership function are a subinterval on [0,1]. Anitha.T, AmarendraBabu.V[2, 3] introduced the notion of vague LI – ideals and vague implicative LI- ideals of lattice implication algebras L.

The object of this paper is to make a study of Interval valued vague ILI – ideals and discuss the properties of Interval valued vague ILI- ideals of lattice implication algebras L.

II. PRELIMINARIES

In this section we collect some important results which were already proved for our use in the next section.

Definition 2.1:[7]Let $(L, \vee, \wedge, ', 0, I)$ be a complemented lattice with the uniTersal bounds 0, I. $\rightarrow$ is another binary operation of L. $(L, \vee, \wedge, \rightarrow, ', 0, I)$ is called a lattice implication algebra, if the following axioms hold, $\forall$ x, y, z $\in$ L,

(I_1) $x\rightarrow (y\rightarrow z) = y\rightarrow (x\rightarrow z)$;

(I_2) $x\rightarrow x = I$;

(I_3) $x\rightarrow y = y'\rightarrow x'$;

(I_4) $x\rightarrow y = y\rightarrow x = I$ implies $x = y$;

(I_5) $(x\rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$;

(L_1) $(x\vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$;

(L_2) $(x\wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$.

Definition 2.2:[7] A lattice implication algebra $(L, \vee, \wedge, \rightarrow, ', 0, I)$ is said to be a lattice H implication algebra if it satisfy the following axiom:$x\vee y \vee ((x \wedge y) \rightarrow z) = I$, $\forall$ x, y, z

Theorem 2.3:[7]Let L be a lattice implication algebra, then for any x, y, z $\in$L, the following conclusions hold:

(1) If $I \to x = I$ then $x = I$;

(2) $I \to x = x$ and $x \to 0 = x$;

(3) $0 \to x = I$ and $x \to I = I$;

(4) $x \le y$ if and only if $x \to y$;

(5) $(x \to z) \to (x \to y) = ((z \wedge x) \to y) = (z \to x) \to (z \to y)$;

(6) $x \to y \le (y \to z) \to (x \to z)$;

(7) $((x \to y) \to y) \to y = x \to y$.

Definition 2.4:[6]Let A be a subset of a lattice implication algebra L. A is said to be an ILI - ideal of if it satisfies the following conditions:

(1) $0 \in A$;

(2) $\forall x, y, z \in L$, $(((x \to y)' \to y)' \to z)' \in A$ and $z \in A$ implies $(x \to y)' \in A$.

Definition2.5: [1]Aninterval valued vague setAin the universe of discourse U is characterized by a truth-membership function$T_G$andfalsemembershipfunction$F_G$given by

$T_A: U \to I[0, 1]$, $F_A: U \to I[0, 1]$

Where$T_A$and$F_A$are set-valued functions on the interval [0,1], respectively. $T_A(z) = [T_A^-(z), T_A^+(z)]$, $T_A^-(z)$ *and* $T_A^+(z)$denote the lower and upper bound on the grade of membership of zderived from "the evidence for z", respectively. Similarly, $F_A(z) = [F_A^-(z), F_A^+(z)]$,$F_A^-(z)$ *and* $F_A^+(z)$denote, respectively, the lower and upper bound on the negation of zderived from "the evidence against z", and $T_A^+(z) + F_A^+(z) \le 1$.

The interval valued vague set G is denoted by $A = \{<z, T_A(z), F_A(z)> / z \in U\}$.

Definition 2.6: Let $A = \{<z, T_A(z), F_A(z)> / z \in U\}$be aninterval vague set of a universe U. For any α, β, t and s $\in$ [0, 1] with $\alpha \le \beta$ and $t \le s$, interval value vague cut of A is a crisp subset $[A_{(\alpha, \beta)}, B_{(t, s)}]$ of the set U given by $[A_{(\alpha, \beta)}, B_{(t, s)}] = \{ x \in U / T_A(x) \ge [\alpha, \beta]$ and $1 - F_A(x) \ge [t, s]\}$.

Definition 2.7: The (α,β)– cut, $A_{(\alpha, \beta)}$ of the interval valued vague set A is the (α, β) – cut of A and hence given by $A_{(\alpha, \beta)} = \{x \in X / T_A(x) \ge [\alpha, \beta]\}$.

Notation: Let I [0, 1] denote the family of all closed subintervals of [0, 1]. If $I_1 = [a_1, b_1]$, $I_2 = [a_2, b_2]$ are two elements of I[0, 1], we call $I_1 \ge I_2$ if $a_1 \ge a_2$ and $b_1 \ge b_2$. We define the term imax to mean the maximum of two interval asimax $[I_1, I_2] = [\max\{a_1, a_2\}, \max\{b_1, b_2\}]$.

Similarly, we can define the term imin of any two intervals.

Definition 2.9: Let A be aninterval valued vague set of a lattice implication algebra L. A is said to be aninterval valued vague LI – ideal (briefly IVVLI – ideal) of L if it satisfies the following conditions:

(1)$T_A(0) \ge T_A(x)$ and $1 - F_A(0) \ge 1 - F_A(x)$ for all $x \in L$,

(2) $T_A(x) \ge \min\{T_A((x \to y)'), T_A(y)\}$ and $1 - F_A(x) \ge \min\{1 - F_A((x \to y)'), 1 - F_A(y)\}$ for all $x, y \in L$.

Definition 2.10: Let A be a interval valued vague set of a lattice implication algebra L. A is said to be an interval valued vague lattice ideal of L if it satisfies the following conditions:

(1) $y \leq x$ then $T_A(x) \geq T_A(y)$, $1- F_A(x) \geq 1- F_A(y)$,

(2) $T_A (x \vee y) \geq \text{imin} \{ T_A (x), T_A (y)\}$ and

$1-F_A (x \vee y) \geq \text{imin} \{ 1-F_A (x), 1-F_A (y)\}$ for $x, y \in L$.

III. Interval Valued Vague ILI– Ideals

Definition 3.1: Let A be a vague set of a lattice implication algebra L. A is said to be aninterval valued vague ILI – ideal (briefly IVVILI – ideal) of L if it satisfies the following conditions:
(1). $T_A(0) \geq T_A(x)$ and $1 - F_A(0) \geq 1 - F_A(x)$ for all $x \in L$.

(2). $T_A((x \to y)') \geq \text{imin}\{T_A(((x \to y)' \to y)'\to z)') , T_A(z)\}$ and

$1 - F_A((x \to y)') \geq \text{imin}\{ 1- F_A(((x \to y)' \to y)'\to z)') , 1- F_A(z)\}$ for all $x, y \in L$.

That is $T_A^+ ((x \to y)') \geq \text{imin}\{T_A^+ (((x \to y)' \to y)'\to z)') ,T_A^+ (z)\}$,

$T_A^- ((x \to y)') \geq \text{imin}\{T_A^- (((x \to y)' \to y)'\to z)') ,T_A^- (z)\}$ and

$1 - F_A^+ ((x \to y)') \geq \text{imin}\{1 - F_A^+ (((x \to y)' \to y)'\to z)') , 1 - F_A^+(z)\}$,

$\mathbf{1}-F_A^- ((x \to y)') \geq \text{imin}\{\mathbf{1}-F_A^- (((x \to y)' \to y)'\to z)') ,\mathbf{1} - F_A^-(z)\}$.

Example 3.2: Let L = {0, a, b, c, d, I} be a set with Cayley table as follows:

→	0	a	b	C	D	I
0	I	I	I	I	I	I
A	c	I	b	C	B	I
B	d	a	I	B	A	I
C	a	a	I	I	A	I
D	b	I	I	B	I	I
I	0	a	b	C	D	I

Define $'$, $\vee$ *and* $\wedge$ operations on L as follows: $x' = x \to 0$, $x \vee y = (x \to y) \to y$, $x \wedge y = ((x' \to y') \to y')'$ for all $x, y \in L$.
Then (L , ∨, ∧ , →, ′, 0,I) is a lattice implication algebra [7]. Define aninterval valued vague set
$A = \{<z,T_A(z), F_A(z) >/ z \in L\}$of L by

A	T_A^+	T_A^-	F_A^+	F_A^-
0	0.7	0.65	0.2	0.18
a	0.7	0.65	0.2	0.18
b	0.5	0.45	0.31	0.22
c	0.5	0.45	0.31	0.22
d	0.7	0.65	0.2	0.18
I	0.5	0.45	0.31	0.22

One can easily verify that A is aIVVILI – ideal of L.

The relation between IVVILI – ideals and IVVLI- ideals of lattice implication algebras is as follows:

Theorem 3.3: AnyIVVILI – ideal of a lattice implication algebra L is anIVVLI – ideal of L.

Proof: Let A be aIVVILI – ideal of a lattice implication algebra L.

Then obviously,$T_A(0) \geq T_A(x)$ and $1 - F_A(0) \geq 1 - F_A(x)$ for all $x \in L$.

Let $x, y, z \in L$, then we have

$T_A((x \to y)') \geq \text{imin}\{T_A(((x \to y)' \to y)'\to z)') , T_A(z)\}$ and

$1 - F_A((x \to y)') \geq \text{imin}\{ 1- F_A(((x \to y)' \to y)'\to z)'), 1- F_A(z)\}$.

Takingy = 0 in the above equation, we obtain

$T_A((x \to 0)') \geq \text{imin}\{T_A(((x \to 0)' \to 0)' \to z)'), T_A(z)\}$

$T_A((x')') = \text{imin}\{T_A(((x')' \to 0)' \to z)'), T_A(z)\}$

$T_A(x) = \text{imin}\{T_A((x \to 0)' \to z)'), T_A(z)\}$

$= \text{imin}\{T_A((x \to z)'), T_A(z)\}$.

and

$1\text{-}F_A((x \to y)') \geq \text{imin}\{1\text{-}F_A(((x \to y)' \to y)'\to z)'), 1\text{-}F_A(z)\}$ and

$1 - F_A((x \to y)') \geq \text{imin}\{ 1- F_A(((x \to y)' \to y)'\to z)'), 1- F_A(z)\}$.

Takingy = 0 in in the above equation, we obtain

$1\text{-}F_A((x \to 0)') \geq \text{imin}\{1\text{-}F_A(((x \to 0)' \to 0)' \to z)'), 1\text{-}F_A(z)\}$

$1\text{-}F_A((x')') = \text{imin}\{1\text{-}F_A(((x')' \to 0)' \to z)'), 1\text{-}F_A(z)\}$

$1\text{-}F_A(x) = \text{imin}\{1\text{-}F_A((x \to 0)' \to z)'), 1\text{-}F_A(z)\}$

$= \text{imin}\{1\text{-}F_A((x \to z)'), 1\text{-}F_A(z)\}$.

Hence A is aIVVLI – ideal of L.

The converse of theorem 3.3 may not be true as seen in the following example:

Example 3.4: Let L be a lattice implication algebra in the example 3.2 and

B = {<z,T_B(z), F_B(z) >/ z ϵ L} is an interval valued vague set as follows:

	T_A^+	T_A^-	F_A^+	F_A^-
0	0.7	0.65	0.2	0.18
a	0.5	0.45	0.31	0.22
b	0.5	0.45	0.31	0.22
c	0.7	0.65	0.2	0.18
d	0.5	0.45	0.31	0.22
I	0.5	0.45	0.31	0.22

Clearly B is anIVVLI – ideal of L. But it is not a IVVILI – ideal of L because

$T_A((a \to b)') \ngeq \text{imin}\{T_A(((a \to b)' \to b)'\to 0)'), T_A(0)\}$ and

$1\text{-}F_A((a \to b)') \ngeq \text{imin}\{1\text{-}F_A(((a \to b)' \to b)'\to 0)'), 1\text{-}F_A(0)\}$

Theorem3.5: In a lattice H implication algebra L, every IVVLI - ideal is a IVVILI – ideal.

Proof: Let A be any IVVLI - ideal of a lattice H implication algebra L.

Then obviously, $T_A(0) \geq T_A(x)$ and $1\text{-}F_A(0) \geq 1\text{-}F_A(x)$all x ϵ L.

We have, $T_A((x \to y)') = T_A((y' \to x')')$

$= T_A((y'\to (y' \to x'))')$

$= T_A((y' \to (x \to y))')$

$= T_A(((x \to y)'\to y)')$

$\geq$ imin{$T_A(((x \to y)' \to y)' \to z)')$, $T_A(z)$}.

and

$1\text{-}F_A((x \to y)') = 1\text{-}F_A((y' \to x')')$

$= 1\text{-}F_A((y' \to (y' \to x'))')$

$= 1\text{-}F_A((y' \to (x \to y))')$

$= 1\text{-}F_A(((x \to y)' \to y)')$

$\geq$ imin{$1\text{-}F_A(((x \to y)' \to y)' \to z)')$, $1\text{-}F_A(z)$}.

Hence A is aIVVILI – ideal of L.

Corolloary3.6: Every IVVILI- ideal A of a lattice implication algebra L is order reversing.

Corolloary3.7: Every IVVILI – ideal of a lattice implication algebra L is aninterval valued vague lattice ideal of L Converse need not to be true.

Remark 3.8: In a lattice H implication algebra L, every interval valued vague lattice ideal is aIVVILI – ideal as seen in the following example.

Example 3.9: Let L = {0, a, b, I} be a set with Cayley table as follows:

→	0	A	b	I
0	I	I	I	I
A	b	I	b	I
B	a	A	I	I
I	0	A	b	I

Define $'$,$\vee$ *and* $\wedge$ –operations on L as follows:

$x' = x \to 0$, $x \vee y = (x \to y) \to y$, $x \wedge y = ((x' \to y') \to y')'$ for all x, y ϵ L.

Then $(L, \vee, \wedge, \to, ', 0, I)$ is a lattice H implication algebra[7]. Let C be aninterval valued vague set in L defined by

C	T_C^+	T_C^-	F_C^+	F_C^-
0	0.7	0.65	0.2	0.18
a	0.5	0.45	0.31	0.22
b	0.5	0.45	0.31	0.22
I	0.5	0.45	0.31	0.22

Clearly, C is bothIVVILI – ideal and interval valued vague lattice ideal of L.

Theorem 3.10: Let A be aninterval valued vague set of a lattice implication algebra L. Then A is a IVVILI – ideal of L if and only if $[A_{(\alpha,\beta)}, B_{(t,s)}]$ is an ILI – ideal of L when $[A_{(\alpha,\beta)}, B_{(t,s)}] \neq \emptyset$, α, β, t and s ϵ [0, 1].

Proof: Assume that A is a IVVILI – ideal of L and α, β, t and s ϵ [0, 1] such that$[A_{(\alpha,\beta)}, B_{(t,s)}] \neq \emptyset$.

Then there exist x ϵ $[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$, and hence $T_A(0) \geq T_A(x) \geq [\,\alpha, \beta]$ and $1\text{-}F_A(0) \geq 1\text{-}F_A(x) \geq [\,t, s]$.

That is 0 ϵ $[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$.

Let x, y , *z* ϵ *L*, if $(((x \to y)' \to y)' \to z)'$ ϵ $[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$and z ϵ$[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$ then

$T_A((((x \to y)' \to y)' \to z)') \geq [\alpha, \beta]$, $T_A(z) \geq [\alpha, \beta]$ and

$$1\text{-}F_A((((x \to y)' \to y)' \to z)') \geq [\alpha, \beta] \text{ , } 1\text{-}F_A(z) \geq [t, s].$$

It follows that

$T_A((x \to y)') \geq \text{imin}\{T_A(((x \to y)' \to y)' \to z)'), T_A(z)\} \geq [\,\alpha, \beta]$ and

$$1\text{-}F_A((x \to y)') \geq \text{imin}\{1\text{-}F_A(((x \to y)' \to y)' \to z)'), 1\text{-}F_A(z)\} \geq [\,t, s],$$

That is $(x \to y)'$ϵ $[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$.

So,$[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$is an ILI ideal of L.

Conversely, Suppose that for any α, β, t and s ϵ [0, 1],$[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}] \neq \emptyset$ is an ILI – ideal of L.

For anyxϵ *L*, x ϵ$[A_{A(x)}, B_{B(x)}]$ and hence $[A_{A(x)}, B_{B(x)}]$ is an ILI ideal of L.

By 0 ϵ$[A_{A(x)}, B_{B(x)}]$it follows that$T_A(0) \geq T_A(x)$ and $1\text{-}F_A(0) \geq 1\text{-}F_A(x)$.

For any x, y , *z* ϵ *L* , let

$[\,\alpha, \beta] = \text{imin}\{T_A((((x \to y)' \to y)' \to z)'), T_A(z)\}$ and

$[\,t, s] = \text{imin}\{1\text{-}F_A((((x \to y)' \to y)' \to z)'), 1\text{-}F_A(z)\}$,

It follows that $[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}] \neq \emptyset$ and hence $[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$ is an ILI – ideal of L.

Since $((x \to y)' \to y)' \to z)'$ ϵ $[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$, z ϵ $[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$this implies$(x \to y)'$ ϵ $[A_{(\alpha\,,\,\beta)}, B_{(t\,,\,s)}]$.

That is$T_A((x \to y)') \geq [\,\alpha, \beta] = \text{imin}\{T_A(((x \to y)' \to y)' \to z)'), T_A(z)\}$ and

$$1\text{-}F_A((x \to y)') \geq [\,t, s] = \text{imin}\{1\text{-}F_A(((x \to y)' \to y)' \to z)'), 1\text{-}F_A(z)\}.$$

So, A is aIVVI LI – ideal of L.

Corollary 3.11: Let A be ainterval valued vague set of a lattice implication algebra L. Then A is a IVVILI – ideal of L if and only if $A_{(\alpha\,,\beta)}$is an ILI – ideal when $A_{(\alpha\,,\beta)} \neq \emptyset$, α, β ϵ [0, 1].

Theorem3.12: (Extension property for IVVILI – ideals) Let A and B be IVVLI- ideals of lattice implication algebra L such that A⊆ B. If A is aIVVILI- ideal of L,then so is B.

Proof: Let Aand B beIVVLI - ideals of lattice implication algebra L such that A ⊆ B.

Since A⊆ B, that is $T_A(x) \leq T_B(x)$ and $1\text{-}F_A(x) \leq 1\text{-}F_B(x)\,\forall$ x ϵ *L*, implies that $A_{(\alpha\,,\beta)} \subseteq B_{(\alpha\,,\beta)}$for every α, β ϵ [0, 1].

If A is a IVVILI then $A_{(\alpha\,,\beta)} \neq \emptyset$ is an ILI – ideal for α, β ϵ [0, 1].

Clearly $B_{(\alpha\,,\beta)} \neq \emptyset$is an ILI – ideal α, β ϵ [0, 1].

It follows B is aIVVILI – ideal of lattice implication algebra L.

IV. CONCLUSION

Since W.L. Gai and D.J. Buehrer proposed the notion of vague sets, these ideas have been applied to various fields. In this paper, Ideas to Lattice implication algebras applied and introduced the notion of IVVILI – ideals. Some properties of IVVILI – ideals are obtained. The relations between

IVVILI - ideals are derived and VLI - ideals, between IVVILI - ideals and its cut sets. This work would serve as a foundation for enriching correspondingmany - valued logical system.

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