Interval Valued Vague ILI – Ideals of Lattice Implication Algebras

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ABSTRACT

The concept of interval valued vague ILI – ideals of lattice implication algebras was introduced. The relationship among interval valued vague ILI – ideals, interval valued vague LI – ideals and interval valued vague lattice ideals was studied. The relation between interval valued vague ILI – ideals and its cut sets was discussed. Extension property of interval valued vague LI – idealsILI – ideal is built.

KEYWORDS- Lattice Implication Algebras, Interval valued vague ILI – ideals, Interval valued vague LI – ideals and Interval valued vague lattice ideals.

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I. INTRODUCTION

In order to research the logical system whose proportional value is given lattice, Y. Xu\(^7\) proposed the concept of lattice implication algebras, and discussed their some properties. Y.Xu, Y.B. Jun and E.H. Roh introduced the notion of LI – ideals of a lattice implication algebras, and discussed their some properties. In particular Young Lin Liu, Yang Xu, Qin and Liu\(^6\) introduced the notion of ILI – ideals of lattice implication algebras.

Vague set theory was first introduced by Gau and Buehrer\(^4\) in 1993. The vague set is an extension of fuzzy set. A vague set H in the universal of discourse U is characterized by a truth membership function \(t_H\) and a false membership function \(f_H\). Actually, vague sets can realistically reflect the actual problem. But more often, the truth-membership and false-membership are in a range. For this reason, the notion of interval valued vague sets was presented by Atanassov in 1989\(^1\). And it is regarded as an extension of the theory of vague sets. In this theory, the truth-membership function and false-membership function are a subinterval on \([0,1]\). Anitha.T, Amarendra Babu.V\(^2,3\) introduced the notion of vague LI – ideals and vague implicative LI- ideals of lattice implication algebras L.

The object of this paper is to make a study of Interval valued vague ILI – ideals and discuss the properties of Interval valued vague ILI- ideals of lattice implication algebras L.

II. PRELIMINARIES

In this section we collect some important results which were already proved for our use in the next section.

**Definition 2.1:** Let \((L,\vee, \wedge, \rightarrow, \neg, 0, 1)\) be a complemented lattice with the universal bounds 0, 1. \(\rightarrow\) is another binary operation of L. \((L, \vee, \wedge, \rightarrow, \neg, 0, 1)\) is called a lattice implication algebra, if the following axioms hold, \(\forall\ x, y, z \in L\),

\[
\begin{align*}
(I_1) & \ x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z); \\
(I_2) & \ x \rightarrow x = I; \\
(I_3) & \ x \rightarrow y = y \rightarrow x; \\
(I_4) & \ x \rightarrow y = y \rightarrow x = I \text{ implies } x = y; \\
(I_5) & \ (x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x; \\
(L_1) & \ (x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z); \\
(L_2) & \ (x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z).
\end{align*}
\]

**Definition 2.2:** A lattice implication algebra \((L, \vee, \wedge, \rightarrow, \neg, 0, 1)\) is said to be a lattice H implication algebra if it satisfy the following axiom:\(x \vee y \vee ((x \wedge y) \rightarrow z) = I, \forall x, y, z\)

**Theorem 2.3:** Let L be a lattice implication algebra, then for any \(x, y, z \in L\), the following conclusions hold:
(1) If \( I \rightarrow x = 1 \) then \( x = I \);
(2) \( I \rightarrow x = x \) and \( x \rightarrow 0 = x \);
(3) \( 0 \rightarrow x = 1 \) and \( x \rightarrow I = I \);
(4) \( x \leq y \) if and only if \( x \rightarrow y \);
(5) \( (x \rightarrow z) \rightarrow (x \rightarrow y) = ((z \land x) \rightarrow y) = (z \rightarrow x) \rightarrow (z \rightarrow y) \);
(6) \( x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z) \);
(7) \( (x \rightarrow y) \rightarrow y = y \rightarrow y \).

**Definition 2.4.** Let \( A \) be a subset of a lattice implication algebra \( L \). \( A \) is said to be an ILI ideal of if it satisfies the following conditions:

1. \( 0 \in A \);
2. \( \forall x, y, z \in L, (((x \rightarrow y) \rightarrow y) \rightarrow z) \in A \) and \( z \in A \) implies \((x \rightarrow y) \rightarrow z) \in A \).

**Definition 2.5.** An interval valued vague set \( A \) in the universe of discourse \( U \) is characterized by a truth-membership function \( T_A \) and false membership function \( F_A \) given by

\[
T_A : U \rightarrow I[0, 1], \quad F_A : U \rightarrow I[0, 1]
\]

Where \( T_A \) and \( F_A \) are set-valued functions on the interval \([0,1]\), respectively. \( T_A(z) = [T_A^L(z), T_A^U(z)] \), \( T_A^L(z) \) and \( T_A^U(z) \) denote the lower and upper bound on the grade of membership of \( z \) derived from “the evidence for \( z \)”, respectively. Similarly, \( F_A(z) = [F_A^L(z), F_A^U(z)] \), \( F_A^L(z) \) and \( F_A^U(z) \) denote, respectively, the lower and upper bound on the negation of \( z \) derived from “the evidence against \( z \)”, and \( T_A^L(z) + F_A^L(z) \leq 1 \).

The interval valued vague set \( G \) is denoted by \( A = \{<z,T_A(z), F_A(z) > | z \in U \} \).

**Definition 2.6.** Let \( A = \{<z,T_A(z), F_A(z) > | z \in U \} \) be an interval vague set of a universe \( U \). For any \( \alpha \), \( \beta \), \( t \) and \( s \in [0,1] \) with \( \alpha \leq \beta \) and \( t \leq s \), interval value vague cut of \( A \) is a crisp subset \( [A(\alpha, \beta), B(t, s)] \) of the set \( U \) given by \( [A(\alpha, \beta), B(t, s)] = \{x \in U / T_A(x) \geq [\alpha, \beta] \) and \( 1 - F_A(x) \geq [t, s] \} \).

**Definition 2.7.** The \((\alpha, \beta)\)–cut, \( A(\alpha, \beta) \) of the interval valued vague set \( A \) is the \((\alpha, \beta)\)–cut of \( A \) and hence given by \( A(\alpha, \beta) = \{x \in X / T_A(x) \geq [\alpha, \beta] \} \).

**Notation:** Let \( I[0,1] \) denote the family of all closed subintervals of \([0,1]\). If \( I_1 = [a_1, b_1], I_2 = [a_2, b_2] \) are two elements of \( I[0,1] \), we call \( I_1 \geq I_2 \) if \( a_1 \geq a_2 \) and \( b_1 \geq b_2 \). We define the term \( \text{imax} \) to mean the maximum of two interval as \( \text{imax} [I_1, I_2] = \{\max \{a_1, a_2\}, \max \{b_1, b_2\}\} \).

Similarly, we can define the term \( \text{imin} \) of any two intervals.

**Definition 2.9.** Let \( A \) be an interval valued vague set of a lattice implication algebra \( L \). \( A \) is said to be an interval valued vague ILI ideal (briefly IVVLI ideal) of \( L \) if it satisfies the following conditions:

1. \( T_A(0) \geq T_A(x) \) and \( 1 - F_A(0) \geq 1 - F_A(x) \) for all \( x \in L \)
2. \( T_A(x) \geq \min \{T_A((x \rightarrow y)\), \( T_A(y)\)\} and \( 1 - F_A(x) \geq \min \{1 - F_A((x \rightarrow y)\), 1 - \( F_A(y)\)\} for all \( x, y \in L \).
**Definition 2.10:** Let $A$ be a interval valued vague set of a lattice implication algebra $L$. $A$ is said to be an interval valued vague lattice ideal of $L$ if it satisfies the following conditions:

1. $y \leq x$ then $T_A(x) \geq T_A(y)$, \ 1- F_A(x) \geq 1- F_A(y),$
2. $T_A(x \lor y) \geq \text{imn} \{T_A(x), T_A(y)\}$ and \ $1 - F_A(x \lor y) \geq \text{imn} \{1 - F_A(x), 1 - F_A(y)\}$ for $x, y \in L$.

**III. INTERVAL VALUED VAGUE ILI – IDEALS**

**Definition 3.1:** Let $A$ be a vague set of a lattice implication algebra $L$. $A$ is said to be an interval valued vague ILI – ideal (briefly IVVILI – ideal) of $L$ if it satisfies the following conditions:

1. $T_A(0) \geq T_A(x)$ and \ $1 - F_A(0) \geq 1 - F_A(x)$ for all $x \in L$.
2. $T_A((x \rightarrow y)') \geq \text{imn} \{T_A(((x \rightarrow y)' \rightarrow (y)' \rightarrow z)'), T_A(z)\}$ and \ $1 - F_A((x \rightarrow y)') \geq \text{imn} \{1 - F_A(((x \rightarrow y)' \rightarrow (y)' \rightarrow z)'), 1 - F_A(z)\}$ for all $x, y \in L$.

That is $T_A^+((x \rightarrow y)') \geq \text{imn} \{T_A^+(((x \rightarrow y)' \rightarrow (y)' \rightarrow z)'), T_A^+(z)\}$, $T_A^-((x \rightarrow y)') \geq \text{imn} \{T_A^-(((x \rightarrow y)' \rightarrow (y)' \rightarrow z)'), T_A^-(z)\}$ and $1 - F_A^+((x \rightarrow y)') \geq \text{imn} \{1 - F_A^+(((x \rightarrow y)' \rightarrow (y)' \rightarrow z)'), 1 - F_A^+(z)\}$, $1 - F_A^-((x \rightarrow y)') \geq \text{imn} \{1 - F_A^-(((x \rightarrow y)' \rightarrow (y)' \rightarrow z)'), 1 - F_A^-(z)\}.$

**Example 3.2:** Let $L = \{0, a, b, c, d, I\}$ be a set with Cayley table as follows:

<table>
<thead>
<tr>
<th>$\rightarrow$</th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>C</th>
<th>D</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>A</td>
<td>c</td>
<td>I</td>
<td>b</td>
<td>C</td>
<td>B</td>
<td>I</td>
</tr>
<tr>
<td>B</td>
<td>d</td>
<td>a</td>
<td>I</td>
<td>B</td>
<td>A</td>
<td>I</td>
</tr>
<tr>
<td>C</td>
<td>a</td>
<td>a</td>
<td>I</td>
<td>I</td>
<td>A</td>
<td>I</td>
</tr>
<tr>
<td>D</td>
<td>b</td>
<td>I</td>
<td>I</td>
<td>B</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>a</td>
<td>b</td>
<td>C</td>
<td>D</td>
<td>I</td>
</tr>
</tbody>
</table>

Define $\rightarrow$, $\lor$ and $\land$ operations on $L$ as follows: $x' = x \rightarrow 0$, $x \lor y = (x \rightarrow y) \rightarrow y$, $x \land y = ((x' \rightarrow y') \rightarrow y')$ for all $x, y \in L$.

Then $(L, \lor, \land, \rightarrow, \land, 0, I)$ is a lattice implication algebra [7]. Define an interval valued vague set $A = \{sz, T_A(z), F_A(z) \rightarrow z \in L\}$ of $L$ by

<table>
<thead>
<tr>
<th>$A$</th>
<th>$T_A^+$</th>
<th>$T_A^-$</th>
<th>$F_A^+$</th>
<th>$F_A^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.7</td>
<td>0.65</td>
<td>0.2</td>
<td>0.18</td>
</tr>
<tr>
<td>a</td>
<td>0.7</td>
<td>0.65</td>
<td>0.2</td>
<td>0.18</td>
</tr>
<tr>
<td>b</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>c</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>d</td>
<td>0.7</td>
<td>0.65</td>
<td>0.2</td>
<td>0.18</td>
</tr>
<tr>
<td>I</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
</tbody>
</table>

One can easily verify that $A$ is an IVVILI – ideal of $L$.

The relation between IVVILI – ideals and IVVLI- ideals of lattice implication algebras is as follows:

**Theorem 3.3:** Any IVVILI – ideal of a lattice implication algebra $L$ is an IVVLI – ideal of $L$.

**Proof:** Let $A$ be an IVVILI – ideal of a lattice implication algebra $L$.

Then obviously, $T_A(0) \geq T_A(x)$ and $1 - F_A(0) \geq 1 - F_A(x)$ for all $x \in L$.

Let $x, y, z \in L$, then we have $T_A((x \rightarrow y)') \geq \text{imn} \{T_A(((x \rightarrow y)' \rightarrow y)' \rightarrow z)'), T_A(z)\}$ and
We have, then obviously, Theorem 3.5: Clearly B is an IVVLI. Let A be any IVVLI ideal of L. But it is not a IVVILI ideal of L because

\[ T_A((a \to b)') \geq \text{imin}\{T_A(((a \to b)') \to b')', T_A(0)\} \]

\[ 1 - F_A((a \to b)') \geq \text{imin}\{1 - F_A(((a \to b)') \to b')', 1 - F_A(0)\} \]

and

\[ 1 - F_A((a \to y)') \geq \text{imin}\{1 - F_A(((a \to y)') \to y')', 1 - F_A(0)\} \]

The converse of theorem 3.3 may not be true as seen in the following example:

**Example 3.4:** Let L be a lattice implication algebra in the example 3.2 and

B = \{<z,T_B(z), F_B(z) >/ z \in L\} is an interval valued vague set as follows:

<table>
<thead>
<tr>
<th>z</th>
<th>( T_A^a )</th>
<th>( T_A^z )</th>
<th>( F_A^a )</th>
<th>( F_A^z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.7</td>
<td>0.65</td>
<td>0.2</td>
<td>0.18</td>
</tr>
<tr>
<td>a</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>b</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>c</td>
<td>0.7</td>
<td>0.65</td>
<td>0.2</td>
<td>0.18</td>
</tr>
<tr>
<td>d</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>l</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Clearly B is an IVVLI ideal of L. But it is not a IVVILI ideal of L because

\[ T_A((a \to b)') \geq \text{imin}\{T_A(((a \to b)') \to b')', T_A(0)\} \] and

\[ 1 - F_A((a \to b)') \geq \text{imin}\{1 - F_A(((a \to b)') \to b')', 1 - F_A(0)\} \]

**Theorem 3.5:** In a lattice H implication algebra L, every IVVLI ideal is a IVVILI ideal.

**Proof:** Let A be any IVVLI ideal of a lattice H implication algebra L.

Then obviously, \( T_A(0) \geq T_A(x) \) and \( 1 - F_A(0) \geq 1 - F_A(x) \) all \( x \in L \).

We have, \( T_A((x \to y)') = T_A((y' \to x')') \)

\[ = T_A((y' \to (y' \to x'))') \]

\[ = T_A((y' \to (x' \to y))') \]

\[ = T_A(((x \to y') \to y')') \]
\[ \geq \text{imin}\{T_A(((x \rightarrow y) \rightarrow y) \rightarrow z') \}, T_A(z)\}. \]

and

\[ 1-F_A((x \rightarrow y')) = 1-F_A((y' \rightarrow x')') \]
\[ = 1-F_A((y' \rightarrow (y' \rightarrow x'))') \]
\[ = 1-F_A((y' \rightarrow (x \rightarrow y'))') \]
\[ = 1-F_A(((x \rightarrow y') \rightarrow y')') \]
\[ \geq \text{imin}\{1-F_A(((x \rightarrow y') \rightarrow y') \rightarrow z'), 1-F_A(z)\}. \]

Hence A is a IVVILI – ideal of L.

**Corollary 3.6:** Every IVVILI- ideal A of a lattice implication algebra L is order reversing.

**Corollary 3.7:** Every IVVILI – ideal of a lattice implication algebra L is an interval valued vague lattice ideal of L. Converse need not to be true.

**Remark 3.8:** In a lattice H implication algebra L, every interval valued vague lattice ideal is a IVVILI – ideal as seen in the following example.

**Example 3.9:** Let L = \{0, a, b, I\} be a set with Cayley table as follows:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>A</th>
<th>b</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>A</td>
<td>b</td>
<td>I</td>
<td>b</td>
<td>I</td>
</tr>
<tr>
<td>B</td>
<td>a</td>
<td>A</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>A</td>
<td>b</td>
<td>I</td>
</tr>
</tbody>
</table>

Define \( \rightarrow, \vee, \wedge, \neg \) operations on L as follows:

\( x' = x \rightarrow 0, x \vee y = (x \rightarrow y) \rightarrow y, x \wedge y = ((x' \rightarrow y') \rightarrow y')' \) for all \( x, y \in L \).

Then \((L, \vee, \wedge, \rightarrow, \neg, 0, I)\) is a lattice H implication algebra. Let C be an interval valued vague set in L defined by

<table>
<thead>
<tr>
<th></th>
<th>( T_C^+ )</th>
<th>( T_C^- )</th>
<th>( F_C^+ )</th>
<th>( F_C^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.7</td>
<td>0.65</td>
<td>0.2</td>
<td>0.18</td>
</tr>
<tr>
<td>a</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>b</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>I</td>
<td>0.5</td>
<td>0.45</td>
<td>0.31</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Clearly, C is both IVVILI – ideal and interval valued vague lattice ideal of L.

**Theorem 3.10:** Let A be an interval valued vague set of a lattice implication algebra L. Then A is an IVVILI – ideal of L if and only if \([A_\alpha, \beta, B_{(t, s)}] \neq \emptyset, \alpha, \beta, t\) and \(s \in [0, 1]\).

**Proof:** Assume that A is a IVVILI – ideal of L and \(\alpha, \beta, t\) and \(s \in [0, 1]\) such that\([A_\alpha, \beta, B_{(t, s)}] \neq \emptyset\).
Then there exist \( x \in [A_{(0, \beta)} B_{(t, s)}] \), and hence \( T_A(0) \geq T_A(x) \geq [\alpha, \beta] \) and \( 1-F_A(0) \geq 1-F_A(x) \geq [t, s] \).

That is \( 0 \in [A_{(0, \beta)} B_{(t, s)}] \).

Let \( x, y, z \in L \), if \( (((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) \in [A_{(0, \beta)} B_{(t, s)}] \) and \( z \in [A_{(0, \beta)} B_{(t, s)}] \) then

\[
T_A(((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) \geq [\alpha, \beta], T_A(z) \geq [\alpha, \beta] \text{ and } 1-F_A(((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) \geq [t, s],
\]

It follows that

\[
T_A ((x \rightarrow y)^{\prime}) \geq \text{ imin} \{T_A(((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) , T_A(z) \} \geq [\alpha, \beta] \text{ and } 1-F_A ((x \rightarrow y)^{\prime}) \geq \text{ imin} \{1-F_A(((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) , 1-F_A(z) \} \geq [t, s],
\]

That is \( (x \rightarrow y)^{\prime} \in [A_{(0, \beta)} B_{(t, s)}] \).

So, \([A_{(0, \beta)} B_{(t, s)}] \) is an ILI ideal of \( L \).

Conversely, Suppose that for any \( \alpha, \beta, t \) and \( s \in [0, 1], [A_{(0, \beta)} B_{(t, s)}] \neq \emptyset \) is an ILI ideal of \( L \).

For any \( x \in [A_{(0, \beta)} B_{(t, s)}] \) and hence \([A_{(0, \beta)} B_{(t, s)}] \) is an ILI ideal of \( L \).

By \( 0 \in [A_{(0, \beta)} B_{(t, s)}] \) it follows that \( T_A(0) \geq T_A(x) \) and \( 1-F_A(0) \geq 1-F_A(x) \).

For any \( x, y, z \in L \), let

\[
[\alpha, \beta] = \text{ imin} \{T_A(((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) , T_A(z) \} \text{ and } [t, s] = \text{ imin} \{1-F_A(((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) , 1-F_A(z) \},
\]

It follows that \([A_{(0, \beta)} B_{(t, s)}] \neq \emptyset \) and hence \([A_{(0, \beta)} B_{(t, s)}] \) is an ILI ideal of \( L \).

Since \( ((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) \in [A_{(0, \beta)} B_{(t, s)}] \) and \( z \in [A_{(0, \beta)} B_{(t, s)}] \), this implies \( (x \rightarrow y)^{\prime} \in [A_{(0, \beta)} B_{(t, s)}] \).

That is \( T_A((x \rightarrow y)^{\prime}) \geq [\alpha, \beta] = \text{ imin} \{T_A(((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) , T_A(z) \} \text{ and } 1-F_A((x \rightarrow y)^{\prime}) \geq [t, s] = \text{ imin} \{1-F_A(((x \rightarrow y)^{\prime} \rightarrow y) \rightarrow z)^{\prime}) , 1-F_A(z) \} \).

So, \( A \) is an IVVILI ideal of \( L \).

**Corollary 3.11:** Let \( A \) be a interval valued vague set of a lattice implication algebra \( L \). Then \( A \) is a IVVILI ideal of \( L \) if and only if \( A_{(0, \beta)} \) is an ILI ideal when \( A_{(0, \beta)} \neq \emptyset, \alpha, \beta \in [0, 1] \).

**Theorem 3.12:** (Extension property for IVVILI ideals) Let \( A \) and \( B \) be IVVILI ideals of lattice implication algebra \( L \) such that \( A \subseteq B \). If \( A \) is an IVVILI ideal of \( L \), then so is \( B \).

**Proof:** Let \( A \) and \( B \) be IVVILI ideals of lattice implication algebra \( L \) such that \( A \subseteq B \).

Since \( A \subseteq B \), that is \( T_A(x) \leq T_B(x) \) and \( 1-F_A(x) \leq 1-F_B(x) \) \( \forall x \in L \), implies that \( A_{(0, \beta)} \subseteq B_{(0, \beta)} \) for every \( \alpha, \beta \in [0, 1] \).

If \( A \) is an IVVILI ideal then \( A_{(0, \beta)} \neq \emptyset \) is an ILI ideal for \( \alpha, \beta \in [0, 1] \).

Clearly \( B_{(0, \beta)} \neq \emptyset \) is an ILI ideal, \( \alpha, \beta \in [0, 1] \).

It follows \( B \) is an IVVILI ideal of lattice implication algebra \( L \).

**IV. CONCLUSION**

Since W.L. Gai and D.J. Buehrer proposed the notion of vague sets, these ideas have been applied to various fields. In this paper, Ideas to Lattice implication algebras applied and introduced the notion of IVVILI ideals. Some properties of IVVILI ideals are obtained. The relations between
IVVILI - ideals are derived and VLI - ideals, between IVVILI - ideals and its cut sets. This work would serve as a foundation for enriching corresponding many-valued logical system.

REFERENCES