A Study on (1,2)*C And (1,2)* C# Sets

Rajkalpana M.¹

¹ Assistant Professor, Sri Krishna Arts and Science College, Coimbatore, Tamil Nadu, India

Email: rajkalpana.m@gmail.com

ABSTRACT

The focus of this paper is to introduce a new class of sets namely (1,2)* C-closed set and (1,2)* C#- closed set in new bitopological setting. Also we investigate some of their properties.

KEYWORDS

(1,2)* bitopology, (1,2)* b-open, (1,2)* semi open, (1,2)* pre open, (1,2)* α-open (1,2)* β-open, (1,2)* regular open, (1,2)* semi regular, (1,2)* C- set, (1,2)* C# - set, Tc space, Tc# space.

*Corresponding author

M. Rajkalpana

Department of mathematics

Sri Krishna Arts And Science College

Coimbatore – 641008

E.mail: rajkalpana.m@gmail.com

Mobile number: 9629807725
INTRODUCTION

The concept of a bitopological space \((X, \tau_1, \tau_2)\) was first introduced by Kelly and the theory has been developed by different mathematician\(^8\). Their attention was mainly confined to the pairwise properties of the two topologies. When the research was going on towards pairwise properties in 1990 the endeavour of Lellis Thivagar brought a new idea on bitopological spaces\(^{10}\). In 2005 Lellis Thivagar and ravi introduced \((1,2)^*\) bitopological space\(^{10}\). The concept of \((1,2)^*\) b-open sets was introduced and studied by Sreeja and Janaki\(^{11}\). The purpose of this paper is to give a new type of open and closed sets namely, \((1,2)^*\) C set, \((1,2)^*\) C\(^\#\) set. Also investigate some of its properties.

LITERATURE REVIEW

The bitopological space \((X, \tau_1, \tau_2)\) was first introduced by Kelly and the theory has been developed by different mathematician\(^8\). Their attention was mainly confined to the pairwise properties of the two topologies. In 1990 the endeavour of Lellis Thivagar brought a new idea on bitopological spaces\(^{10}\). In 2005 Lellis Thivagar and ravi introduced \((1,2)^*\) bitopological space\(^{10}\). The concept of \((1,2)^*\) b-open sets was introduced and studied by Sreeja and Janaki\(^{11}\). The purpose of this paper is to give a new type of open and closed sets namely, \((1,2)^*\) C set, \((1,2)^*\) C\(^\#\) set. Also investigate some of its properties.

PRELIMINARIES

**Definition 1.2.1**

Let \((X, \tau_1, \tau_2)\) be a bitopological space. A subset \(A\) of \(X\) is said to be \((1,2)^*\) b-open if
\[ A \subseteq (\tau_{1,2} - \text{int}(\tau_{1,2} - \text{cl}(A))) \cup (\tau_{1,2} - \text{cl}(\tau_{1,2} - \text{int}(A))). \]
It is denoted by \((1,2)^*\) bo\((X)\).

**Definition 1.2.2**

A subset \(S\) of a bitopological space \((X, \tau_1, \tau_2)\) is said to be \(\tau_{1,2}\)-open if \(S = A \cup B\)
where \(A \in \tau_1\) and \(B \in \tau_2\).

**Definition 1.2.3**

A subset \(S\) of a bitopological space \((X, \tau_1, \tau_2)\) is said to be \(\tau_{1,2}\)-closed if the complement of \(S\) is \(\tau_{1,2}\)-open.

**Definition 1.2.4**

A subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\) is called
\[
(1,2)^* \text{ semi open if } A \subseteq \tau_{1,2} - \text{cl}(\tau_{1,2} - \text{int}(A))
\]
\[
(1,2)^* \text{ pre open if } A \subseteq \tau_{1,2} - \text{int}(\tau_{1,2} - \text{cl}(A))
\]
(1, 2)° α-open if \(A \subseteq \tau_{1,2} \cap \text{int}(\tau_{1,2} \cap \text{cl}(A))\)

(1, 2)° β-open if \(A \subseteq \tau_{1,2} \cap \text{cl}(\tau_{1,2} \cap \text{int}(A))\)

(1, 2)° regular open if \(A = \tau_{1,2} \cap \text{int}(\tau_{1,2} \cap \text{cl}(A))\)

(1, 2)° semi regular if \(A\) is both (1, 2)° semi open and (1, 2)° semi closed.

(1, 2)° generalized closed if \(\tau_{1,2} \cap \text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_{1,2} \cap \text{open}\) in \(X\).

(1, 2)° semi generalized closed if \(\tau_{1,2} \cap \text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_{1,2} \cap \text{open}\) in \(X\).

(1, 2)° α generalized closed if \(\tau_{1,2} \cap \alpha \cap \text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_{1,2} \cap \text{open}\) in \(X\).

(1, 2)° generalized α- closed if \(\tau_{1,2} \cap \alpha \cap \text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_{1,2} \cap \text{open}\) in \(X\).

(1, 2)° generalized semi closed if \(\tau_{1,2} \cap \text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is semi open in \(X\).

**Definition 1.2.6**

A bitopological space \((X, \tau_1, \tau_2)\) is called

(1, 2)° semi T₀ space if for any two distinct points \(x, y\) in \(X\) there exists a (1, 2)° semi open set containing one but not the other.

(1, 2)° Tₜ₀ space if every (1, 2)° GS closed set is \(\tau_{1,2} \cap \text{closed}\)

(1, 2)° α- space if every (1, 2)° α- closed set is \(\tau_{1,2} \cap \text{closed}\).

(1, 2)° αTₜ₀ space if every (1, 2)° αG- closed set is \(\tau_{1,2} \cap \text{closed}\).

**MAIN WORK**

**(1, 2)° C-Closed Sets And (1, 2)° C°- Closed Sets**

**Definition 2.1**

A subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\) is called (1, 2)° C-closed if \(\tau_{1,2} \cap \text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is (1, 2)° b-open in \((X, \tau_1, \tau_2)\).

The complement of a (1, 2)° C-closed set is called (1, 2)° C-open.

**Definition 2.2**

A subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\) is called (1, 2)° C°- closed if \(\tau_{1,2} \cap \alpha \cap \text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is (1, 2)° C°- open in \((X, \tau_1, \tau_2)\).

The complement of a (1, 2)° C°-closed set is called (1, 2)° C°-open.
Theorem 2.3
(i) Every $\tau_{1,2}$-closed set is $(1,2)^*C$-closed.
(ii) Every $\tau_{1,2}$-regular closed set is $(1,2)^*C$-closed.
(iii) Every $\tau_{1,2}$-closed set is $(1,2)^*C^\#$-closed.
(iv) Every $(1,2)^*\alpha$-closed set is $(1,2)^*C^\#$-closed.
(v) Every $(1,2)^*C^\#$-closed set is $(1,2)^*\alpha g$-closed.
(vi) Every $(1,2)^*C^\#$-closed set is $(1,2)^*gs$-closed.

Proof
(i) Suppose $U$ is $(1,2)^*b$-open set such that $A \subseteq U$. Since $A$ is $\tau_{1,2}$-closed, $\tau_{1,2}-\text{cl}(A) \subseteq U$. Hence $A$ is $(1,2)^*C$-closed.
(ii) Suppose $U$ is $(1,2)^*b$-open set such that $A \subseteq U$. Since $A$ is $\tau_{1,2}$-regular closed, $\tau_{1,2}-\text{Cl}(\text{int}(A)) = A \subseteq U$. Hence $A$ is $(1,2)^*C$-closed.
(iii) Suppose $U$ is $(1,2)^*C$-open set such that $A \subseteq U$. Since $A$ is $\tau_{1,2}$-closed, $\tau_{1,2}-\text{cl}(A) = A \subseteq U$. We know that $\tau_{1,2}-\alpha\text{cl}(A) \subseteq \tau_{1,2}-\text{cl}(A) \subseteq U$. Thus $A$ is $(1,2)^*C^\#$-closed.
(iv) Suppose $U$ is $(1,2)^*C$-open set such that $A \subseteq U$. Let $A$ be $(1,2)^*\alpha$-closed set. Therefore $\tau_{1,2}-\alpha\text{cl}(A) = A \subseteq U$. Hence $A$ is $(1,2)^*C^\#$-closed.
(v) Suppose $U$ is $\tau_{1,2}$-open set such that $A \subseteq U$. Since $A$ is $(1,2)^*C^\#$-closed set, $\tau_{1,2}-\alpha\text{cl}(A) \subseteq U$. We know that every $\tau_{1,2}$-open is $(1,2)^*C$-open set. Hence $A$ is $(1,2)^*\alpha g$-closed.
(vi) Suppose $U$ is $(1,2)^*\tau_{1,2}$-open set such that $A \subseteq U$. Let $A$ be $(1,2)^*C^\#$-closed set. Then $\tau_{1,2}-\alpha\text{cl}(A) \subseteq U$. Since $\tau_{1,2}-\text{ scl}(A) \subseteq \tau_{1,2}-\alpha\text{cl}(A) \subseteq U$. Hence $A$ is $(1,2)^*gs$-closed.

Remark 2.4
However the converse of the above theorem need not be true may be seen by the following examples.

Example
$X = \{ a, b, c \}$, $\tau_1 = \{ \phi, \{ a, b \}, X \}$, $\tau_2 = \{ \phi, \{ a, c \}, X \}$. $(1, 2)^*C^\#$-closed sets= $\{ \phi, \{ b \}, \{ c \}, \{ b, c \}, X \}$. Here $\{ b, c \}$ is $(1,2)^*C^\#$-closed set but not $(1,2)^*\alpha$-closed and $\tau_{1,2}$-closed.
Because closure and alpha closure of $\{ b, c \}$ is not equal to $\{ b, c \}$.

$X = \{ a, b, c \}$, $\tau_1 = \{ \phi, \{ a \}, X \}$, $\tau_2 = \{ \phi, \{ b \}, X \}$. $(1, 2)^*C^\#$-closed sets= $\{ \phi, \{ c \}, \{ a, c \}, \{ b, c \}, X \}$. Here $\{ b \}$ and $\{ a \}$ are $(1,2)^*gs$-closed set but not $(1,2)^*C^\#$-closed set.

The above results as shown by the following diagram
1. (1,2)* $C$-closed, 2. $\tau_{1,2}$-closed, 3. (1,2)* $C^\#$-closed, 4. (1,2)* $\alpha$-closed, 5. (1,2)* $ag$-closed, 6. (1,2)* $gs$-closed. 7. $\tau_{1,2}$-regular closed.

Remark 2.5

The union and intersection of two (1,2)* $C^\#$-closed sets need not be (1,2)* $C^\#$-closed set as shown in the following example.

**Example**

$X = \{ a, b, c \}$, $\tau_1 = \{ \phi, \{ a, b \}, X \}$, $\tau_2 = \{ \phi, \{ c \}, \{ b, c \}, \{ a, c \}, X \}$, (1, 2)* $C^\#$-closed sets= $\{ \phi, \{ a \}, \{ b \}, \{ c \}, \{ a, b \}, \{ a, c \}, X \}$. Here $\{ b \}$ and $\{ c \}$ are (1,2)* $C^\#$-closed set but $\{ b, c \}$ is not (1,2)* $C^\#$-closed set.

$X = \{ a, b, c \}$, $\tau_1 = \{ \phi, \{ a \}, X \}$, $\tau_2 = \{ \phi, X \}$, (1, 2)* $C^\#$-closed sets= $\{ \phi, \{ b \}, \{ c \}, \{ a, b \}, \{ b, c \}, \{ a, c \}, X \}$. Here $\{ a, b \}$ and $\{ a, c \}$ are (1,2)* $C^\#$-closed set but $\{ a \}$ is not (1,2)* $C^\#$-closed set.

**Theorem 2.6**

If a set $A$ is (1,2)* $C^\#$-closed then (1,2)* $\alpha cl(A)$-A contains no nonempty $\tau_{1,2}$-closed set.

**Proof**

Let $F$ be a $\tau_{1,2}$-closed subset of (1,2)* $\alpha cl(A)$-A. Therefore $A \subseteq F^C$ and $F \subseteq (1,2)^* \alpha cl(A)$. $F^C$ is $\tau_{1,2}$-open. Since every $\tau_{1,2}$-open set is (1,2)* $C$-open, $F^C$ is (1,2)* $C$-open. Let $A$ be (1,2)* $C^\#$-closed. Then (1,2)* $\alpha cl(A) \subseteq F^C$ whenever $A \subseteq F^C$. Thus $F \subseteq [(1,2)^* \alpha cl(A)]^C$. Thus $F \subseteq [(1,2)^* \alpha cl(A)] \cap [(1,2)^* \alpha cl(A)]^C$. Hence $F = \phi$.

**Theorem 2.7**

If a set $A$ is (1,2)* $C^\#$-closed then (1,2)* $\alpha cl(A)$-A contains no nonempty C-closed set.

**Proof**
Let $F$ be a $(1,2)^*\alpha\text{cl}(A) - A$. Therefore $F \subseteq \tau_{1,2}-\text{cl}(A) - A$ and $A \subseteq F^C$ and $F^C$ is $(1,2)^*\text{C-open}$. Since $A$ is $(1,2)^*\text{C#-closed}$ set, $(1,2)^*\text{acl}(A) \subseteq F^C$ whenever $A \subseteq F^C$. This implies that $F \subseteq [(1,2)^*\text{acl}(A)]^C$. Thus $F \subseteq [ (1,2)^*\text{acl}(A) ] \cap [ (1,2)^*\text{acl}(A) ]^C$, Hence $F = \emptyset$.

**Theorem 2.8**

If $A$ is a $(1,2)^*\text{C-open}$ and a $(1,2)^*\text{C#-closed}$ subset of $(X, \tau_{1,2})$ then $A$ is a $(1,2)^*\alpha$-closed subset of $(X, \tau_{1,2})$.

**Proof**

Let $A$ be $(1,2)^*\text{C-open}$ and a $(1,2)^*\text{C#-closed}$ subset of $(X, \tau_{1,2})$. Therefore $\tau_{1,2}-\text{cl}(A) \subseteq A$. We know that $A \subseteq \tau_{1,2}-\text{cl}(A)$. This implies that $\tau_{1,2}-\text{cl}(A) = A$. Hence $A$ is a $(1,2)^*\alpha$-closed subset of $(X, \tau_{1,2})$.

**Theorem 2.9**

Let $A$ be $(1,2)^*\text{C#-closed}$ subset of $(X, \tau_{1,2})$ if $A \subseteq B \subseteq (1,2)^*\text{cl}(A)$ then $B$ is also a $(1,2)^*\text{C#-closed}$ subset of $(X, \tau_{1,2})$.

**Proof**

Suppose $U$ is $(1,2)^*\text{C-open}$ such that $B \subseteq U$. Let $A \subseteq B \subseteq U$. Then $A \subseteq U$. Since $A$ is $(1,2)^*\text{C#-closed}$ set, $\tau_{1,2}-\text{cl}(A) \subseteq U$. But $A \subseteq B \subseteq (1,2)^*\alpha-\text{cl}(A)$. Therefore $(1,2)^*\alpha-\text{cl}(A) \subseteq (1,2)^*\alpha-\text{cl}(B)$. Hence $(1,2)^*\alpha-\text{cl}(B) \subseteq U$. Thus $B$ is also a $(1,2)^*\text{C#-closed}$ subset of $(X, \tau_{1,2})$.

**Theorem 2.10**

For each $a \in X$ either $\{a\}$ is $(1,2)^*\text{C-closed}$ or $\{a\}^C$ is $(1,2)^*\text{C#-closed}$.

**Proof**

Suppose $\{a\}$ is not $(1,2)^*\text{C-closed}$ in $X$. Then $\{a\}^C$ is not $(1,2)^*\text{C-open}$. Therefore the only $(1,2)^*\text{C-open}$ set containing $\{a\}^C$ is $X$ and $(1,2)^*\text{cl}(\{a\}^C) \subseteq X$. Hence $\{a\}^C$ is $(1,2)^*\text{C#-closed}$ set.

**Theorem 2.11**

Let $A$ be $(1,2)^*\text{C#-closed}$ in $X$ then $A$ is $(1,2)^*\alpha$-closed if and only if $(1,2)^*\text{acl}(A) - A$ is $\tau_{1,2}$-closed.

**Proof**

Suppose $A$ is $(1,2)^*\alpha$-closed. Then $A = (1,2)^*\alpha-\text{cl}(A)$. Therefore $(1,2)^*\alpha-\text{cl}(A) - A = \emptyset$. Hence $(1,2)^*\text{cl}(A) - A$ is $\tau_{1,2}$-closed.
Conversely, Suppose \((1,2)^*\alpha\text{-cl}(A) = \tau\) closed. Let \(A\) be \((1,2)^*\alpha\text{-c}\)-closed in \(X\). By the Theorem 2.6 \((1,2)^*\alpha\text{-cl}(A) = \phi\). Then \((1,2)^*\alpha\text{-cl}(A) = A\). Hence \(A\) is \((1,2)^*\alpha\)-closed.

**Remark 2.12**

For any subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\)
\((1,2)^*\alpha\text{-cl}(A^C) = [ (1,2)^*\alpha\text{-int}(A) ]^C\).

**Theorem 2.13**

A subset \(A\) of \((X, \tau_1, \tau_2)\) is \((1,2)^*\alpha\text{-c}\)-open if and only if \(F \subseteq (1,2)^*\alpha\text{-int}(A)\) whenever \(F\) is \((1,2)^*\alpha\text{-cl}\) closed and \(F \subseteq A\).

**Proof**

Let \(F \subseteq A\). Then \(A^C \subseteq F^C\) and \(F^C\) is \((1,2)^*\alpha\text{-c}\)-open. Since \(A^C\) is \((1,2)^*\alpha\text{-c}\)-closed, \((1,2)^*\alpha\text{-cl}(A^C) \subseteq F^C\). By using the Remark 2.12 \([ (1,2)^*\alpha\text{-int}(A) ]^C \subseteq F^C\). Hence \(F \subseteq (1,2)^*\alpha\text{-int}(A)\).

Conversely, Let \(A^C \subseteq U\) where \(U\) is \((1,2)^*\alpha\text{-c}\)-open. Then \(U^C \subseteq A\) where \(U^C\) is \((1,2)^*\alpha\text{-c}\)-closed. By hypothesis \(U^C \subseteq (1,2)^*\alpha\text{-int}(A)\). Therefore \([ (1,2)^*\alpha\text{-int}(A) ]^C \subseteq U\). By the Remark 2.12 \((1,2)^*\alpha\text{-cl}(A^C) \subseteq U\). Hence \(A^C\) is \((1,2)^*\alpha\text{-c}\)-closed. Thus \(A\) is \((1,2)^*\alpha\text{-c}\)-open.

**Theorem 2.14**

If \((1,2)^*\alpha\text{-int}(A) \subseteq B \subseteq A\) and \(A\) is \((1,2)^*\alpha\text{-c}\)-open then \(B\) is also \((1,2)^*\alpha\text{-c}\)-open.

**Proof**

Let \((1,2)^*\alpha\text{-int}(A) \subseteq B \subseteq A\). This implies that \(A^C \subseteq B^C \subseteq [ (1,2)^*\alpha\text{-int}(A) ]^C\). By the Remark 2.12 \(A^C \subseteq B^C \subseteq (1,2)^*\alpha\text{-cl}(A^C)\). Also \(A^C\) is \((1,2)^*\alpha\text{-c}\)-closed. By the Theorem 2.9 \(B^C\) is also \((1,2)^*\alpha\text{-c}\)-closed. Hence \(B\) is \((1,2)^*\alpha\text{-c}\)-open.

**Remark 2.15**

Every \(\tau_{1,2}\) open set is \((1,2)^*\alpha\text{-c}\)-open. But the converse may not be true as shown in the following example.

**Example**

Let \(X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, X\}, \tau_2 = \{\phi, \{a, c\}, X\}, \tau_{1,2}\) open set = \(\phi, \{a, b\}, \{a, c\}, X\). \((1,2)^*\alpha\text{-c}\)-open set = \(\phi, \{a\}, \{a, b\}, \{a, c\}, X\). Here \(\{a\}\) is \((1,2)^*\alpha\text{-c}\)-open set but not \((1,2)^*\alpha\text{-c}\)-open.

**Definition 2.16**

A space \((X, \tau_1, \tau_2)\) is called a \((1,2)^*\alpha\text{-c}\)-space if every \((1,2)^*\alpha\text{-c}\)-closed set in it is \((1,2)^*\alpha\text{-c}\)-closed.

**Theorem 2.17**
Every \((1,2)^* \text{C}^\#\)-closed set is \((1,2)^* \alpha\)-closed in \((1,2)^* \text{T}_1\) space.

**Proof**

Let \((X, \tau_1, \tau_2)\) be \((1,2)^* \text{T}_1\) space and \(A\) be \((1,2)^* \text{C}^\#\)-closed set. Therefore for every \(x \in A\) there exists a \(\tau_{1,2}\)-open set \(U_x\) such that \(x \in U_x\) and \(y \notin U_x\). Then \(\bigcup_{x \in A} U_x = U\) is \(\tau_{1,2}\) – open. Therefore \(U\) is \((1,2)^* \text{C}\) – open also \(A \subseteq U\) and \(y \notin U\). Since \(A\) is \((1,2)^* \text{C}^\#\)-closed set, \(\tau_{1,2} - \text{acl}(A) \subseteq U\) whenever \(A \subseteq U\). This implies that \(y \notin \tau_{1,2} - \text{acl}(A)\). Then \(\tau_{1,2} - \text{acl}(A) \subseteq A\). This implies that \(A = \tau_{1,2} - \text{acl}(A)\). Hence \(A\) is \((1,2)^* \alpha\)-closed.

**Theorem 2.18**

For a space \((X, \tau_1, \tau_2)\) the following conditions are equivalent.

(i) \((X, \tau_1, \tau_2)\) is a \((1,2)^* \text{T}_C^\#\) space.

(ii) Every singleton subset of \((X, \tau_1, \tau_2)\) is either \((1,2)^* \text{C}\)-closed or \((1,2)^* \alpha\)-open.

**Proof**

(i) \(\rightarrow\) (ii) Let \(x \in X\). Suppose \(\{x\}\) is not \((1,2)^* \text{C}\)-closed subset of \((X, \tau_1, \tau_2)\). Then \(X - \{x\}\) is not a \((1,2)^* \text{C}\)-open set. So \(X\) is only \((1,2)^* \text{C}\)-open set containing \(X - \{x\}\). So \(X-\{x\}\) is a \((1,2)^* \text{C}\)-closed subset of \((X, \tau_1, \tau_2)\). Let \((X, \tau_1, \tau_2)\) be \((1,2)^* \text{T}_C^\#\) space. Then \(X-\{x\}\) is a \((1,2)^* \alpha\)-closed subset of \((X, \tau_1, \tau_2)\). Hence \(\{x\}\) is a \((1,2)^* \alpha\)-open subset of \((X, \tau_1, \tau_2)\).

(ii) \(\rightarrow\) (i) Let \(A\) be a \((1,2)^* \text{C}^\#\)-closed set of \(X\). Trivially \(A \subseteq (1,2)^* \text{acl}(A)\). Let \(x \in (1,2)^* \text{acl}(A)\). By (ii) \(\{x\}\) is either \((1,2)^* \text{C}\)-closed or \((1,2)^* \alpha\)-open.

Case - A

\(\{x\}\) is \((1,2)^* \text{C}\)-closed. If \(x \notin A\), then \((1,2)^* - \text{acl}(A) - A\) contains a nonempty \((1,2)^* \text{C}\)-closed set \(\{x\}\). By theorem 2.7, we arrive at a contradiction. Thus \(x \in A\).

Case – B

\(\{x\}\) is \((1,2)^* - \alpha\)-open. Since \(x \in (1,2)^* - \text{acl}(A)\), \(\{x\} \cap A \neq \emptyset\). This implies that \(x \in A\). So \((1,2)^* - \text{acl}(A) \subseteq A\). Therefore \((1,2)^* - \text{acl}(A) = A\). Then \(A\) is \((1,2)^* - \alpha\)-closed. Hence \((X, \tau_1, \tau_2)\) is a \((1,2)^* \text{T}_C^\#\) space.

**Theorem 2.19**

Every \((1,2)^* \text{T}_b\) space is a \((1,2)^* \text{T}_C^\#\) space.

**Proof**

Let \(A\) be a \((1,2)^* \text{C}^\#\)-closed set. Then by the Theorem 2.3, \(A\) is \((1,2)^* - \text{gs}\)-closed. Since \((X, \tau_1, \tau_2)\) is a \((1,2)^* - \text{T}_b\) space, \(A\) is \(\tau_{1,2}\)-closed. It is true that every \(\tau_{1,2}\)-closed set is \((1,2)^* - \alpha\)-closed. Therefore \(X\) is a \((1,2)^* \text{T}_C^\#\) space.

**Remark 2.20**
The converse of above theorem need not be true may be seen in the following example.

**Example**

\[ X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, X\}, \tau_2 = \{\phi, \{b\}, X\}, \quad (1,2)^* C^\# -\text{closed} \quad \text{sets} = \{\phi, \{c\}, \{a,c\}, \{b,c\}, X\} \]. Here all \((1,2)^* C^\#\)-closed sets are \((1,2)^* \alpha\)-closed. Therefore \(X\) is \((1,2)^* T_C^\#\) space. But it is not \((1,2)^* T_b\) space because \(\{b\}\) is not \(\tau_{1,2}\)-closed.

**Theorem 2.21**

Every \((1,2)^* T_b\) space is a \((1,2)^* T_C^\#\) space.

**Proof**

Let \(A\) be a \((1,2)^* C^\#\)-closed set. Then by the Theorem 2.3, \(A\) is \((1,2)^* \alpha\)-closed. Since \((X, \tau_1, \tau_2)\) is a \((1,2)^* \alpha T_b\) space, \(A\) is \(\tau_{1,2}\)-closed. It is true that every \(\tau_{1,2}\)-closed set is \((1,2)^* \alpha\)-closed. Therefore \(X\) is a \((1,2)^* T_C^\#\) space.

**Definition 2.22**

A bitopological space \((X, \tau_1, \tau_2)\) is called a \((1,2)^* T_C^\#\) space if every \((1,2)^* C^\#\)-closed set in it is \(\tau_{1,2}\)-closed.

**Theorem 2.23**

Let \((X, \tau_1, \tau_2)\) be a bitopological space. If a set \(A\) is \((1,2)^* C^\#\)-closed then \(\tau_{1,2}\)-cl\(A\) contains no non empty \((1,2)^* b\)-closed set.

**Proof**

Suppose \(\tau_{1,2}\)-cl\(A\) contains \((1,2)^* b\)-closed set \(F\). Then \(A \subseteq F^C\). \(F^C\) is \((1,2)^* b\)-open and \(A\) is \((1,2)^* C\)-closed. Therefore \(\tau_{1,2}\)-cl\(A\) \(\subseteq F^C\). Then \(F \subseteq [\tau_{1,2}\text{-cl}(A)]^C\). Hence \(F \subseteq [\tau_{1,2}\text{-cl}(A)] \cap [\tau_{1,2}\text{-cl}(A)]^C = \phi\). This implies that \(F = \phi\).

**Theorem 2.24**

For a bitopological space \((X, \tau_1, \tau_2)\) the following condition are Equivalent.

(i) \((X, \tau_1, \tau_2)\) is a \((1,2)^* T_C^\#\) space.

(ii) Every singleton subset of \((X, \tau_1, \tau_2)\) is either \((1,2)^* b\)-closed or \(\tau_{1,2}\)-open.

**Proof**

(i)\(\rightarrow\)(ii) Let \(x \in X\). Suppose \(\{x\}\) is not \((1,2)^* b\)-closed subset of \((X, \tau_1, \tau_2)\). Then \(X - \{x\}\) is not a \((1,2)^* b\)-open set. So \(X\) is only \((1,2)^* b\)-open set containing \(X - \{x\}\). So \(X - \{x\}\) is a \((1,2)^* C\)-closed subset of \((X, \tau_1, \tau_2)\). Since \((X, \tau_1, \tau_2)\) is a \((1,2)^* T_C^\#\) space. Then \(X - \{x\}\) is a \(\tau_{1,2}\)-closed. Hence \(\{x\}\) is \(\tau_{1,2}\)-open.
Let \( A \) be a \((1,2)^* \) \( C \)-closed subset of \( X \). Trivially \( A \subseteq \tau_{1,2} \cdot \text{cl}(A) \). Let \( x \in \tau_{1,2} \cdot \text{cl}(A) \).

By (ii) \( \{x\} \) is either \((1,2)^* \) \( b \)-closed or \( \tau_{1,2} \)-open.

**Case - A**

\( \{x\} \) is \((1,2)^* \) \( b \)-closed. If \( x \notin A \), then \( \tau_{1,2} \cdot \text{cl}(A) \setminus A \) contains a nonempty \((1,2)^* \) \( b \)-closed set \( \{x\} \). By the Theorem 2.23, we arrive at a contradiction. Thus \( x \in A \).

**Case - B**

Suppose that \( \{x\} \) is \( \tau_{1,2} \)-open. Since \( x \in \tau_{1,2} \cdot \text{cl}(A) \), \( \{x\} \cap A \neq \emptyset \). This implies that \( x \in A \). So \( \tau_{1,2} \cdot \text{cl}(A) \subseteq A \). Therefore \( \tau_{1,2} \cdot \text{cl}(A) = A \). Then \( A \) is \( \tau_{1,2} \)-closed. Hence \((X, \tau_1, \tau_2)\) is a \((1,2)^* \) \( T_C \) space.

**Remark 2.25**

\((1,2)^* \) \( T_C^\# \) spaces and \((1,2)^* \) \( T_C \) space are independent of one another as the following example shows.

**Example**

\[ X = \{a, b, c\}, \tau_1 = \{\emptyset, \{a\}, X\}, \tau_2 = \{\emptyset, \{a\}, \{b, c\}, X\}, \]
\[ (1,2)^* \text{-closed sets} = \{\emptyset, \{a\}, \{b, c\}, X\}, \]
\[ (1,2)^* \text{-closed sets} = \{\emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\} \].

Here all \((1,2)^* \text{-closed sets} = \tau_{1,2} \)-closed. So \((X, \tau_1, \tau_2)\) is a \((1,2)^* \) \( T_C \) space. But not \((1,2)^* \) \( T_C^\# \) space. Because the set \( \{a, c\} \) is not \( \tau_{1,2} \)-closed.

**CONCLUSION**

In this study we discussed about two types of sets namely \((1,2)^* \text{-C-sed set} \) and \((1,2)^* \text{-C}\#-set \) in new bitopological setting and two type of spaces, \((1,2)^* \) \( T_C^\# \) spaces and \((1,2)^* \) \( T_C \) space are introduced. Also, some of their properties are investigated with some examples.

**REFERENCES**

6. C.E. Aull and W.J. Thorn, Separation axioms between $T_0$ and $T_1$ space, Indag math, 1962; 24: 26-37


